MEASUREMENT 1

MEASUREMENT OF REVOLUTION NUMBER, MOMENT OF INERTIA AND FRICTION TORQUE

1. Introduction

The aim of this measurement is to determine the relationship between the ***n*** revolution number and the elapsed time ***t*** (***n(t)*** diagram) from the instant of switching off the electric motor until the complete standstill. A further task is the determination of the M_f friction torque which slows down the rotation of the shaft as a function of revolution number. In order to finish the tasks successfully, it is necessary to get familiar with the basic definitions (revolution number, moment of inertia, ***n(t)*** diagram) and their measurement methods.

THE SHORT DESCRIPTION OF THE MEASUREMENT: To determine the $M_f(n)$ friction torque of the electric motor depending on the revolution number, the ***n(t)*** diagram is needed. After switching off the electric motor, the rotor of the motor continuously slows down due to the friction torque, and finally it stops. The ***n(t)*** function can be determined with the help of a revolution number transducer mounted on the rotor shaft and with a computer.

Then the friction torque can be determined at any point of the ***n(t)*** diagram using Newton's second law:

$$M_f = \Theta\varepsilon,$$

where ε is the angular deceleration obtained from the ***n(t)*** diagram, and Θ is the moment of inertia, which is measured with the help of another electric motor, which is the same type as the one used for the ***n(t)*** diagram.

SUMMARY:

- Measurement of ***n(t)*** diagram and Θ moment of inertia. These measurements can be performed parallel.
- Determination of ε angular deceleration with the help of ***n(t)*** diagram.
- Calculation of M_f friction torque.

2. The *n(t)* diagram and its measurement

To keep the rotating part of the motor at constant speed, the driving motor must cover the braking torques. In our case this braking torque is coming from the friction of the bearings (the braking torque of the air resistance acting on the surface of the rotor is neglected).

When the motor is switched off, the tested machine part will rotate with a decreasing speed due to the friction until it stops.

If the instantaneous value of the ***n*** revolution number is measured during the decelerating rotation, and ***n*** is plotted against the time, the ***n=n(t)*** diagram can be obtained. The ***n(t)*** diagram shows the revolution number of the shaft as a function of time.

The measurement setup is shown in Figure 1. The shaft of the electric motor "1" is connected to the „2” revolution number transducer, which is giving a voltage signal proportional to the revolution speed. Its analogue signal is processed by the computer "3". The main steps of signal processing are digitizing, filtering and plotting as a function of time. During the measurement, the motor is switched off and we wait until it stops (approx. 35 s). Then, the ***n=n(t)*** plot appears in the screen.

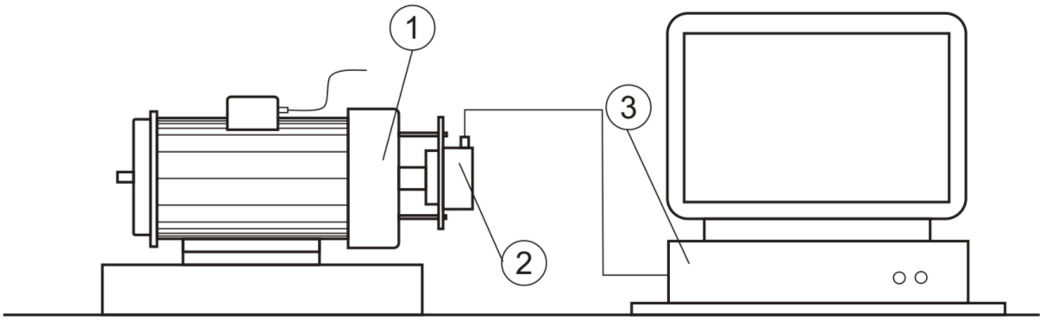

Figure 1. Schematic drawing of the setup for measuring n(t) diagram
(Parts of the rig: 1: electric motor; 2: revolution number transducer*; 3: computer)*

A typical shape of ***n(t)*** function can be seen on Figure 2. We can see, that the rotating part stops at time t_0 after the motor was switched off. With the help of the measurement recording program, the time ***t***, the revolution number ***n*** and the angular deceleration ε can be read at any arbitrary point (the angular acceleration is calculated automatically by the program).

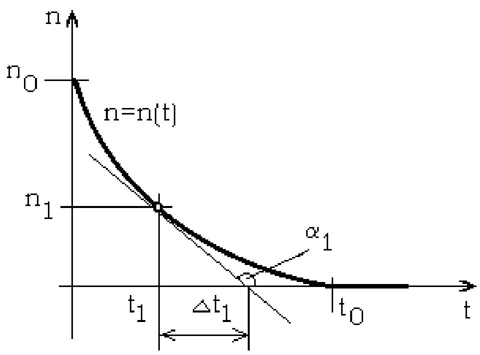

Figure 2. ***n(t)*** diagram

The ***n(t)*** diagram obtained during the measurement has to be drawn on a plotting paper with the help of the uniformly distributed 12 measuring points read by the program. (The number of points may vary depending on the group size) After the measurement, the measurement points are written on the blackboard so the ***n(t)*** diagram can be drawn immediately.

3. The moment of inertia, and its measurement

3.1. The moment of inertia

Let us consider the case when a ring of mass ***m*** with radius ***R*** moves on a circular path around its own axis (e.g. bicycle wheel which spokes have negligible mass). Then its moment of inertia is

$$\Theta = mR^2.$$

The moment of inertia of a full cylinder with respect to its own rotation axis is

$$\Theta = \frac{1}{2} mR^2.$$

where ***m*** is the mass and ***R*** is the radius of the cylinder. Generally, the moment of inertia depends on the shape of the rotating body, the mass distribution and the relative position of the axis of rotation. It can be determined using mathematical tools. If an irregularly shaped body is considered, we divide it into small masses Δm_i, each with a r_i radius from the axis of rotation. Then the moment of the inertia of a small part is

$$\Delta\Theta_i = \Delta m_i r_i^2.$$

The moment of inertia of the full body can be calculated as a sum of these small parts:

$$\Theta = \int r^2\, dm \cong \sum \Delta m_i r_i^2.$$

It is important to note that, the different moments of inertia with respect to the same axis are additive. It means, that for example if a point mass m_2 on a radius r_2 is mounted on a cylinder with m_1 mass and r_1 radius ($r_2 < r_1$), the resulting moment of inertia is

$$\Theta = \frac{1}{2} m_1 r_1^2 + m_2 r_2^2.$$

3.2. The moment of inertia of the rotor

To evaluate our measurements, the moment of inertia of the rotor (Θ) is needed. According to the left-hand side of **Figure 3** we can turn our rotor into a physical pendulum by mounting an additional mass (cylinder of uniform mass-distribution) on the rotating part at a distance of ***e*** from the rotation axis.

The physical pendulum is a rigid body having extension and hanging on a rotation axis ***(A)***, see middle part of **Figure 3**. Later, the rotor-additional-mass pendulum will be referred to, as physical pendulum. The additional mass is a solid cylinder with ***d=2R*** diameter, and its axis is at a distance ***e*** from the rotation axis. Let us neglect the mass of the metallic plate helping the mounting of the cylinder to the rotor, therefore the metal plate has no role in the moment of inertia calculation. Due to the additional mass, the centre of gravity of the physical pendulum is displaced from point ***A*** to point ***S***.

The moment of inertia of the additional mass consists of the $\frac{1}{2}m_a r^2$ value, calculated respect to the cylinder axis, and from the $m_a e^2$ so called Steiner part (the Steiner-law was explained in the lecture) coming from the fact that the cylinder axis is displaced from the rotation axis by a distance of ***e***. Then, as a result

$$\Theta_a = \frac{1}{2}m_a r^2 + m_a e^2.$$

The resulting moment of inertia of the rotor-additional-mass physical pendulum (right hand side of **Figure 3**.) is the sum of the moments of inertia of the rotor and the additional mass:

$$\Theta_p = \Theta + \Theta_a.$$

If the moment of inertia of the physical pendulum could be determined, then Θ could be obtained easily, because

$$\Theta = \Theta_p - \Theta_a = \Theta_p - \frac{1}{2}m_a r^2 - m_a e^2.$$

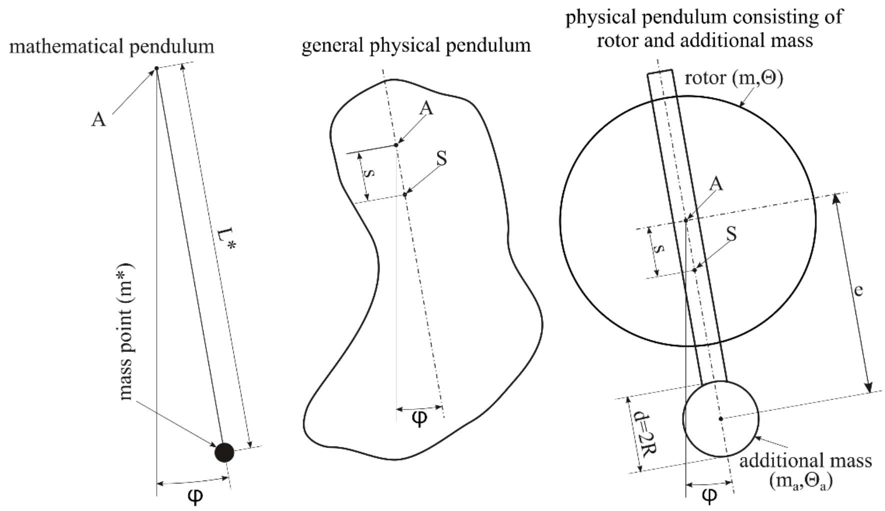

Figure 3. The comparison of the physical and the mathematical pendulum

Because of the fact, that the period time $\boldsymbol{T_p}$ of the oscillation of the physical pendulum depends on the moment of inertia, the measurement of the period time of the oscillation is a way to obtain Θ. The problem is that there is no formula for the calculation of the time period in the case of a complex geometry. Thus, the physical pendulum is compared with an equivalent mathematical pendulum; see the left-hand side of **Figure 3**. The ***L**** reduced length of the mathematical pendulum must be chosen so that the $\boldsymbol{\varepsilon_p}$ angular acceleration of the physical pendulum is the same as $\boldsymbol{\varepsilon_m}$ angular acceleration of the equivalent mathematical pendulum. In this case the period times will be the same. Applying Newton's second law, the angular accelerations are

$$\varepsilon_p = \frac{M_p}{\Theta_p} = -\frac{(m + m_a) \cdot g \cdot s \cdot \sin\varphi}{\Theta_p},$$

$$\varepsilon_{\mathrm{m}} = \frac{M_m}{\Theta_{\mathrm{m}}} = -\frac{m^* \cdot g \cdot L^* \cdot \sin\varphi}{m^* {L^*}^2} = -\frac{g \cdot \sin\varphi}{L^*}.$$

From the equality of the two angular accelerations, the reduced length is

$$L^* = \frac{\Theta_{\mathrm{p}}}{(m + m_a) \cdot s}.$$

It is worth to note that there are only the parameters of the physical pendulum in the left-hand side of the equation.

Based on the equality of the period times and the formula of period time of mathematical pendulum, the following expression can be written:

$$T = T_m = 2\pi\sqrt{\frac{L^*}{g}} = T_p = 2\pi\sqrt{\frac{\Theta_{\mathrm{p}}}{(m + m_a) \cdot s \cdot g}}.$$

Since it is difficult to calculate with the ***s*** distance between the ***S*** centre of gravity and point ***A***, the torque equilibrium on the point ***A*** is applied (***e*** length can be measured):

$$(m + m_a) \cdot s \cdot g = m_a \cdot e \cdot g.$$

Finally, the moment of inertia of the physical pendulum can be determined by measuring the oscillation time and the parameters of the additional mass:

$$\Theta_{\mathrm{p}} = m_a \cdot e \cdot g \cdot \left(\frac{T}{2\pi}\right)^2.$$

3.3. The process of measuring the moment of inertia

The measurement will be carried out by groups with 3 members. Each group will measure the rotors moment of inertia independently, which means 4 values in case of 4 groups. It will be seen, that the values fluctuate, therefore the 4 values will be averaged and the $\boldsymbol{\Theta_M}$ mean value will be used to calculate the friction torque.

This means that as long as all the groups do not measure the moment of inertia, we cannot calculate the friction torque.

Figure 4 shows the measure rig and its schematic drawing. It can be seen that the additional mass "2" (see **Figure 3**) is attached to the motor shaft.

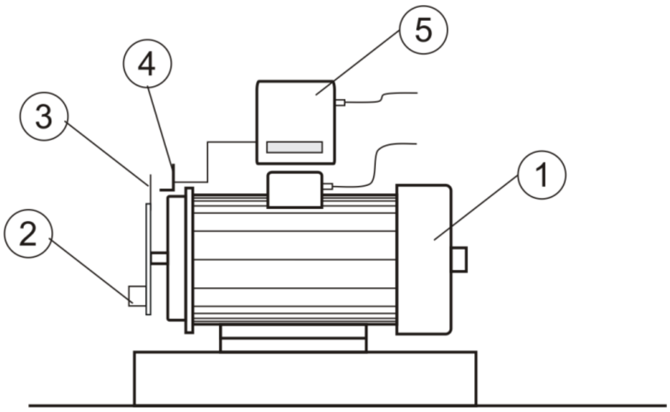

Figure 4. Measurement of moment of inertia

(Above: Parts of the measurement rig:1: electric motor; 2: additional mass; 3: plate; 4: magnetic impulse sender; 5: period time measuring device

Below: screen of the period time measuring device)

Each group selects a specific additional mass and writes down its parameters (diameter $\boldsymbol{d}$, mass $\boldsymbol{m_a}$). The additional mass is fixed to the bored metal plate by a screw. Remember to measure the distance $\boldsymbol{e}$ before fixing. The assembled physical pendulum is rotated until the mark scratched on the plate "3" is vertical (5-degree rotation).

The magnet on the disc passes before the "4" magnetic impulse sender, which gives a signal, taking into account the direction of the passage (always one per oscillation). From this, the period of the oscillation can be measured directly and can be read from the display “5”. Knowing the time $\boldsymbol{T}$, the moment of inertia can be calculated as above.

4. Friction torque

When all groups had completed the moment of inertia measurement and had obtained the $\boldsymbol{\Theta_M}$ mean moment of inertia, furthermore everybody is ready with the drawing of $\boldsymbol{n(t)}$ diagram, then the measurement can be continued by calculating and depicting the friction torque on a separate plotting paper.

Of course, you do not have to calculate the friction torque at all 12 measurement points. Before starting the exercise, everyone will get a serial number and will only work with the data corresponding to the appropriate number from the table drawn on the blackboard. Remember, at the beginning of the lab exercise we make the $\boldsymbol{n(t)}$ diagram and record 12 uniformly distributed measurement points on the blackboard including the time $\boldsymbol{t}$, the revolution number $\boldsymbol{n}$ and the angular acceleration $\boldsymbol{\varepsilon}$.

The friction torque can be determined using Newton's II. law:

$$M_f = \Theta_M \varepsilon.$$

After calculating the result, the obtained values are written into the table drawn onto the blackboard.

4.1. Checking angular acceleration

Check at each measurement point (denote the time by $\boldsymbol{t_1}$ and the corresponding revolution number by $\boldsymbol{n_1}$) by drawing a tangent line at $\boldsymbol{t_1}$ in the $\boldsymbol{n=n(t)}$ diagram and calculating the slope of that line (see **Figure 2**). The slope of the tangent line gives the value of the angular acceleration at time $\boldsymbol{t_1}$ (it is negative because it slows down):

$$\varepsilon_1 \approx \frac{\Delta\omega}{\Delta t} = \frac{2\pi \cdot \Delta n}{\Delta t} = -2\pi \frac{n_1}{\Delta t_1} = -2\pi \cdot \tan \alpha_1,$$

where

$$\tan \alpha_1 = \frac{n_1}{\Delta t_1}.$$

In the above formula the $\boldsymbol{\omega=2\pi n}$ relationship between angular velocity and revolution number was used. Furthermore, we applied the fact that the angular acceleration is equal to the change of the angular velocity per unit time ($\boldsymbol{\varepsilon\approx\Delta\omega/\Delta t}$). The angular acceleration calculated from the slope and readings from the program should be nearly the same!

Preparation to the measurement

- Bring 2 pieces of plotting paper of A4 size, pencil, ruler, calculator.
- Before the measurement, we shall check the proper preparation for the measurement with a small entry test (sample questions are on the website) Please note that there may be other questions in the test.
- Filling the lab report templates at home up to point 4 (points 5-8 will be detailed in the lesson).

Remarks on the measurement description or on the whole measurement are welcome to the e-mail address csizmadia@hds.bme.hu

Appendix

6. Revolution number and its measurement

6.1. The revolution number of the rotating shaft is the number of revolutions per unit time. Its notation is mostly ***n***. Its dimension is *rev/s* or *rev/min* (*rpm*), but often only *1/min* is used. The relationship between ***ω*** angular velocity and the ***n*** revolution number is

$$\omega = 2 \cdot \pi \cdot n \left[\frac{rad}{s} \right].$$

From the point of view of the measuring concept the instruments measuring the revolution number can be divided into two groups:

- **speed indicators:** measure the average revolution number
- **tachometers:** measure the instantaneous revolution number

6.2. Speed indicators

a.) Measurements of small revolution numbers (up to 120-150 1/min) can be made with a stopwatch and by counting revolutions with naked eye. When the mark on the rotating machine shaft gets to a certain place, we start the stopwatch and synchronously start to count from 0. Having measured time ***t*** of some (***N***) revolutions, the number of revolutions per unit time (***n***) can be determined as:

$$n = \frac{N}{t} \left[\frac{1}{s}, \frac{1}{\min} \right].$$

If the speed fluctuates, the latter relationship gives the average revolution number for the measured interval. The longer the time for counting the number of revolutions, the more accurately the revolution number can be determined.

b.) For higher speed of rotation, a special counting device must be used. One of the simplest of these is the so called **jumping-figure speed counter**. The rotating shaft of this device turns gears. One of them completes one revolution while the other rotates only 1/10, and so on. Reading the numbers uniformly painted on the cylinder jacket we get the number of revolutions. Such a device is used in kilowatt-hour meters, tape recorders, speedometers of cars etc. If the elapsed time is measured with a stopwatch, the average revolution number can be calculated.

c.) **Speed indicator working with clockworks** (**Jacquet indicator, Figure 5.)** counts the revolutions only for a fixed time, generally for 6 seconds. The time measuring device of the instrument connects its pointer with the shaft of the instrument which joins the rotating machine part for 6 seconds after pressing the starting bottom. After these six seconds there is no more connection which means the end of the measurement is at the same time. On the dial the tenfold value of the counted revolutions i.e. rpm. (revolution per minute) can be read. This device is the so-called Jacquet indicator. (Order of measurement: press the rubber joint of the instrument to the shaft of the machine to establish connection between them, press the start button, release start bottom, wait around 8 seconds while the device measures, release the connection between the shafts, read the value). By pressing the start button, the instrument is reset and when it is released the counting and the clockwork starts. Therefore, the buttons should be released only when the rubber joint piece rotates together with the shaft! It is advisable to reset the instrument only when starting the new measurement.

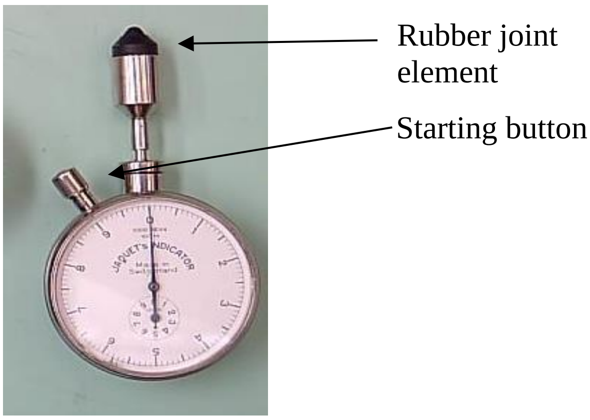

Figure 5. Jacquet indicator.

d.) **Electronic speed indicators** are composed of one or more markers giving a voltage pulse and of a pulse counter operating on electronic principle. The marker is generally a photocell in front of which a disc with slits ensures one or more illuminations at each turn. The electric circuit is closed by the illuminated photocell. (**Figure 6**)

This counter can be placed far off the site of the measurement.

In order to increase accuracy and reduce measurement time it is advisable to increase the number of slits.

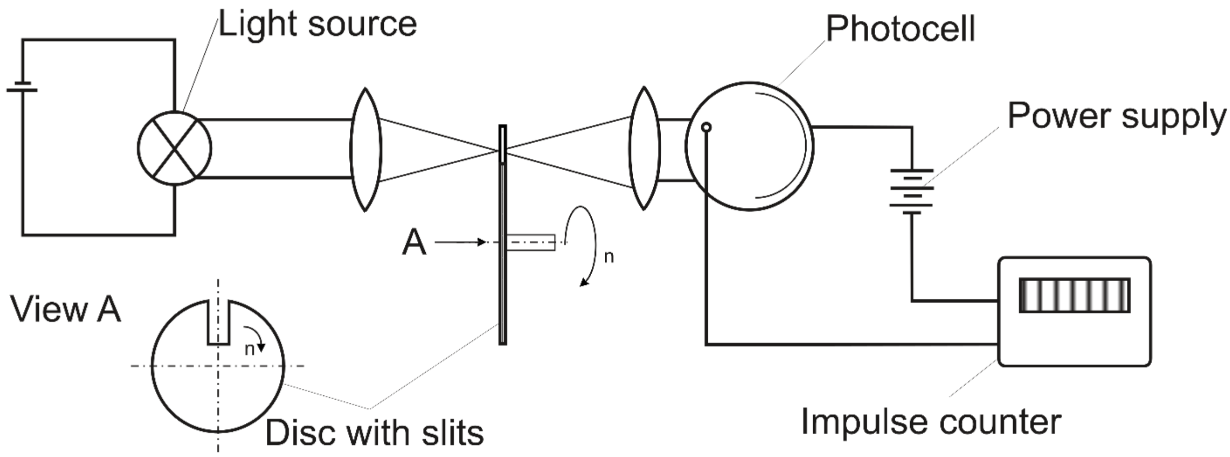

Figure 6. Electronic speed indicator

e.) **Electronic stroboscopes** illuminate the rotating or vibrating body with flashing lamps of variable and adjustable frequencies. If the frequency of the flashes corresponds to the speed of the rotating object, the object seems to be stationary. The frequency i.e. the revolution number can be read on the instrument. (**Figure. 7.)**

Figure 7. Stroboscope.

6.3. The most well-known **tachometer** is the dynamo, which works on an electronic principle. This instrument produces voltage signal proportional to the revolution number. If the relationship between the voltage and revolution number is known (calibration), then the revolution number can be determined from the measured voltage signal.