THEORETICAL INVESTIGATION OF THE TWO PHASE FLOW IN A DIFFUSER BUILT IN A PNEUMATIC CONVEYING PIPE

László Kovács – Sándor Váradi

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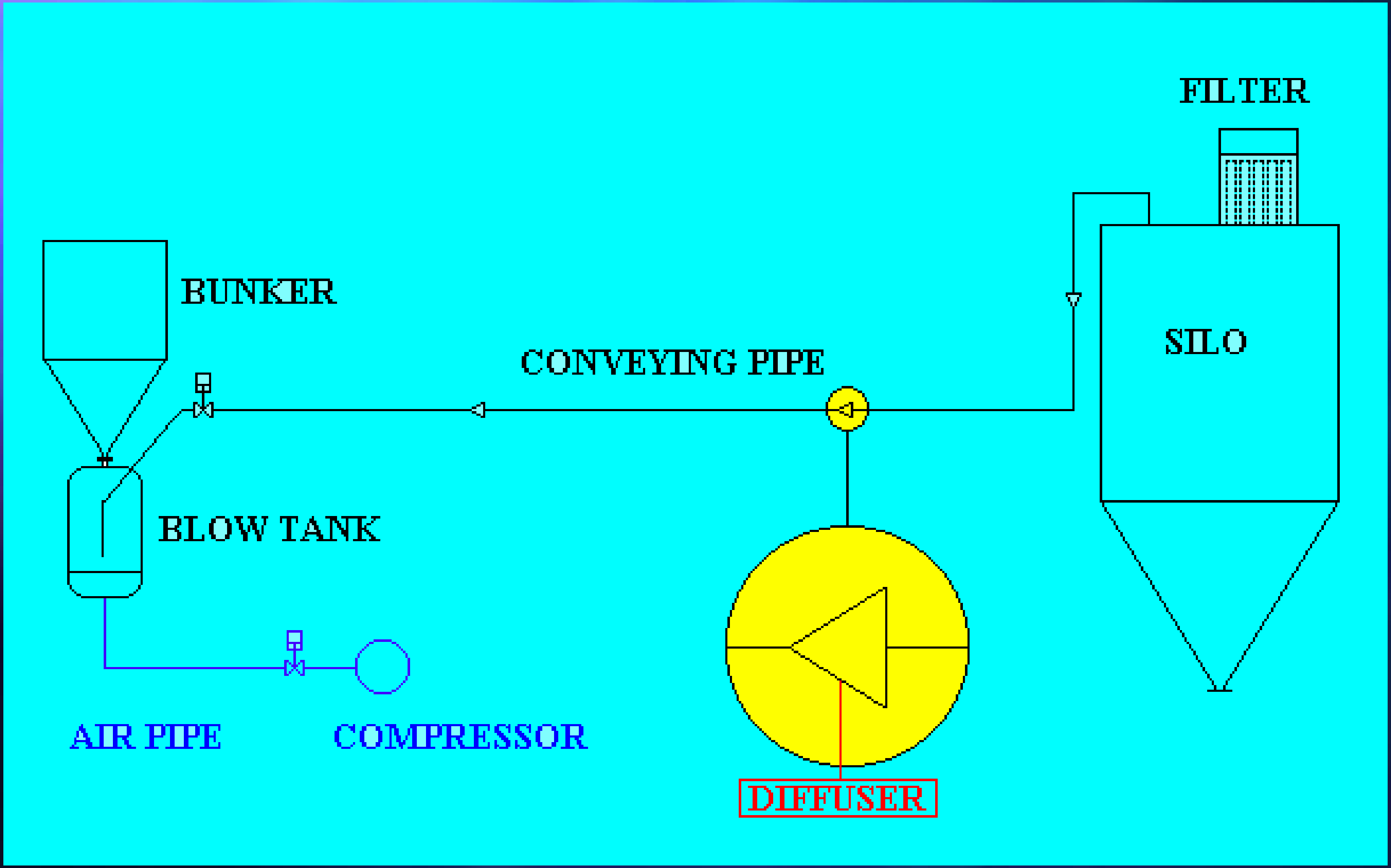
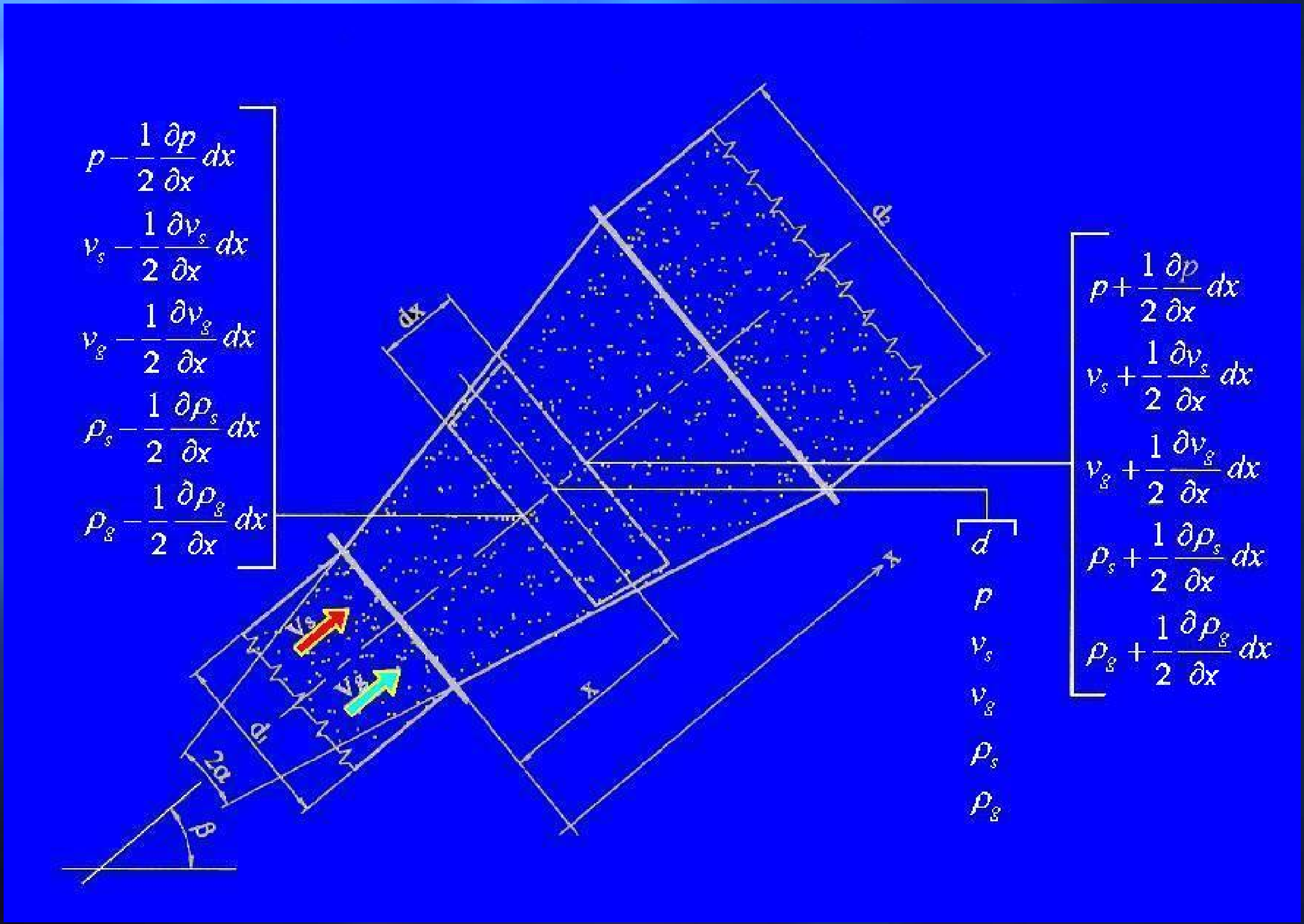
Application in a long distance pneumatic conveying pipe



Diffuser built in conveying pipeline. Nomenclature in the control volume



Continuity equation for the conveying gas

$$\left(A-\frac{dA}{2}\right)\left(\rho_g-\frac{d\rho_g}{2}\right)\left(v_g-\frac{dv_g}{2}\right)-\left(A+\frac{dA}{2}\right)\left(\rho_g+\frac{d\rho_g}{2}\right)\left(v_g+\frac{dv_g}{2}\right)=0$$

$$A-\frac{dA}{2}=\frac{\pi}{4}\left(d^2-2\,d\,tg\alpha\,dx\right)$$

$$A+\frac{dA}{2}=\frac{\pi}{4}\left(d^2+2\,d\,tg\alpha\,dx\right)$$

$$\frac{dp}{dx}=-\frac{p}{v_g}\frac{dv_g}{dx}-\frac{p}{d}4\,tg\alpha$$

Continuity equation for the solid particle

$$\left(A-\frac{dA}{2}\right)\left(\rho_s-\frac{d\rho_s}{2}\right)\left(v_s-\frac{dv_s}{2}\right)-\left(A+\frac{dA}{2}\right)\left(\rho_s+\frac{d\rho_s}{2}\right)\left(v_s+\frac{dv_s}{2}\right)=0$$

$$\frac{d\rho_s}{dx}=-\frac{\rho_s}{v_s}\frac{dv_s}{dx}-\frac{\rho_s}{d}4\,tg\,\alpha$$

Momentum equation for the conveying gas

$$-\left(A-\frac{dA}{2}\right)\left(\rho_g-\frac{d\rho_g}{2}\right)\left(v_g-\frac{dv_g}{2}\right)^2+\left(A+\frac{dA}{2}\right)\left(\rho_g+\frac{d\rho_g}{2}\right)\left(v_g+\frac{dv_g}{2}\right)^2=$$

$$=\left(A-\frac{dA}{2}\right)\left(p-\frac{dp}{2}\right)-\left(A+\frac{dA}{2}\right)\left(p+\frac{dp}{2}\right)+\left[\left(A+\frac{dA}{2}\right)-\left(A-\frac{dA}{2}\right)\right]p-$$

$$-\ dF_D\ -\ dm_s g\left(\sin\beta+\mu\cos\beta\right)$$

Elementary forward moving force

$$dF_D = \frac{d^2\pi}{4}\frac{\rho_s dx}{m_1}\frac{\rho_g}{2}A_o C_D\left(v_g - v_s\right)^2$$

- After transforming and rearranging

$$\frac{dv_g}{dx} = \frac{\rho_{go} v_g^2}{\rho_{go} v_g^2 - p_o}\left[\frac{4 p_o tg\alpha}{\rho_{go} v_g d} - \frac{\rho_s A_o C_D}{2 m_1 v_g}\left(v_g - v_s\right)^2 - \frac{\rho_s g\, p_o}{\rho_{go} p\, v_g}\left(\sin\beta + \mu\cos\beta\right)\right]$$

The drag coefficient is given by Kaskas

$$C_D = \frac{24}{\mathrm{Re}} + \frac{4}{\sqrt{\mathrm{Re}}} + 0.4$$

- **The Reynolds number:**

$$\mathrm{Re} = \frac{d_p \left(v_g - v_s\right)}{\upsilon_g}$$

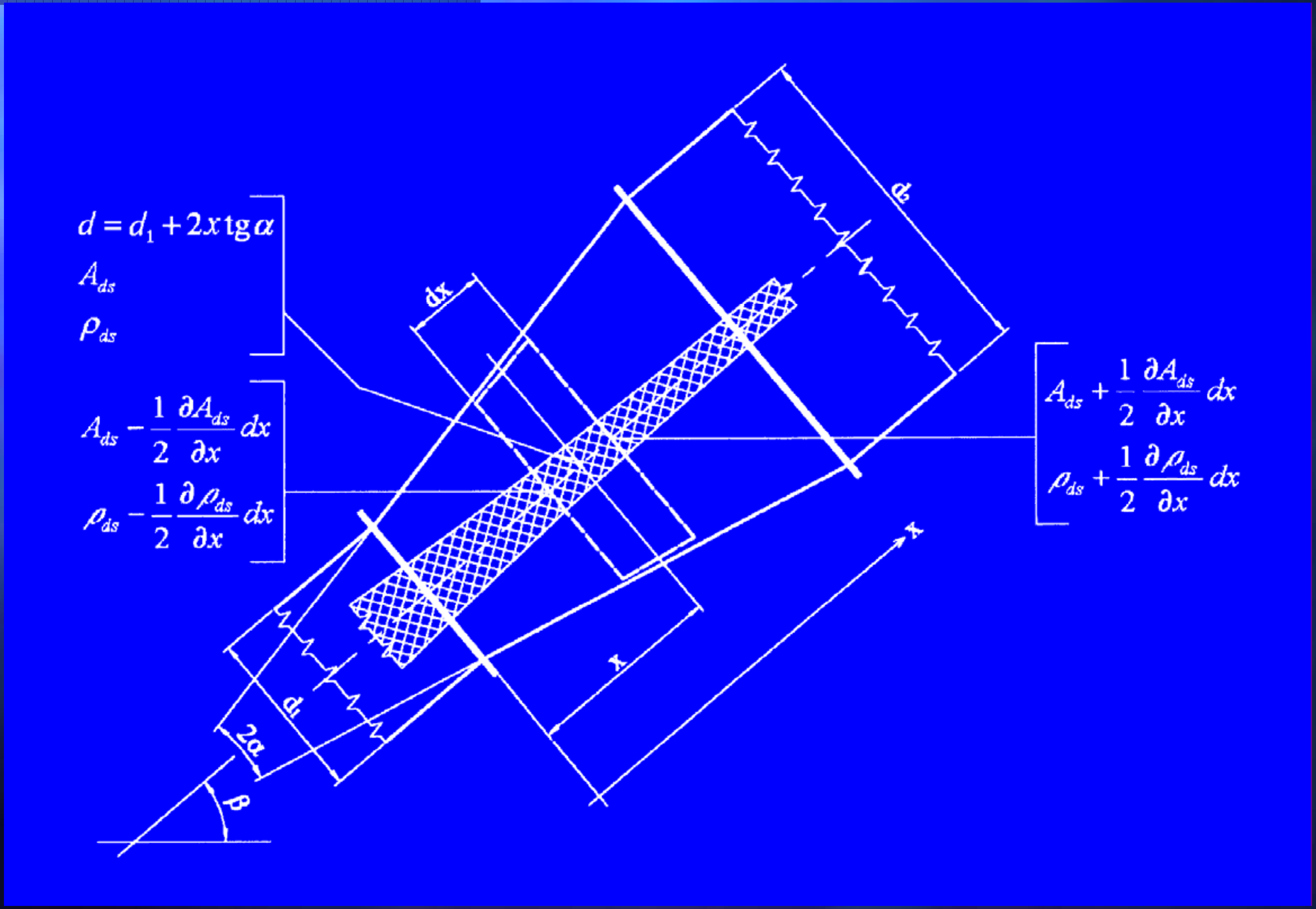
Momentum equation for the solid particle

$$-\left(A-\frac{dA}{2}\right)\left(\rho_s-\frac{d\rho_s}{2}\right)\left(v_s-\frac{dv_s}{2}\right)^2+\left(A+\frac{dA}{2}\right)\left(\rho_s+\frac{d\rho_s}{2}\right)\left(v_s+\frac{dv_s}{2}\right)^2=$$

$$=-dF_D-dm_s g\left(\sin\beta+\mu\cos\beta\right)$$

$$\frac{dv_s}{dx}=\frac{\rho_{go}\,p\,A_o C_D}{2m_1 p_o v_s}\left(v_g-v_s\right)^2-\frac{g}{v_s}\left(\sin\beta+\mu\cos\beta\right)$$

Cross section tightening in the diffuser created by the flowing material



Cross section of the dense material

- **At the point x**

$$A_{ds|x} = \frac{\rho_s}{\rho_{ds}} A = \frac{\rho_s}{\rho_{ds}} \left(d_1 + 2\,x\,tg\,\alpha \right)^2 \frac{\pi}{4}$$

- **At the point x-dx/2**

$$A_{ds|x-dx/2} = A_{ds|x} - \frac{1}{2}\, d\, A_{ds}$$

The cross section depends on concentration and on the local point

$$A_{ds} = f\left(\rho_s ; x\right)$$

$$d\,A_{ds} = \frac{\partial\,A_{ds}}{\partial\,\rho_s}\,d\,\rho_s + \frac{\partial\,A_{ds}}{\partial x}\,dx =$$

$$= \frac{\pi}{4}\left[\frac{\left(d_1 + 2\,x\,tg\,\alpha\right)^2}{\rho_{ds}}\,d\,\rho_s + \frac{\rho_s}{\rho_{ds}}\,2\left(d_1 + 2\,x\,tg\alpha\right)2\,tg\alpha\,dx\right]$$

Considering the above mentioned Eqs., we can obtain the following equation:

$$A_{ds|x-dx/2} = \frac{\rho_s}{\rho_{ds}}\left(d_1 + 2\,x\,tg\alpha\right)^2 \frac{\pi}{4} - \frac{1}{2}\left[\frac{\pi}{4}\frac{\left(d_1 + 2\,x\,tg\alpha\right)^2}{\rho_{ds}}\,d\,\rho_s + \frac{\pi}{4}\frac{\rho_s}{\rho_{ds}}2\left(d_1 + 2\,x\,tg\alpha\right)2\,tg\alpha\,dx\right] =$$

$$= \frac{\pi}{4}\left[\frac{\rho_s}{\rho_{ds}}\left(d^2 - 2\,d\,tg\alpha\,dx\right) - \frac{d^2}{2\,\rho_{ds}}\,d\,\rho_s\right]$$

The cross section of the dense material at the point *x-dx/2*

$$A_{ds|x+dx/2} = \frac{\pi}{4}\left[\frac{\rho_s}{\rho_{ds}}\left(d^2 + 2\,d\;tg\alpha\;dx\right) + \frac{d^2}{2\,\rho_{ds}}\,d\,\rho_s\right]$$

- The free cross section for gas

$$A^*_{x-dx/2} = A_{x-dx/2} - A_{ds|x-dx/2} = \frac{\pi}{4}\left(d^2 - 2\,d\;tg\alpha\;dx\right) - \frac{\pi}{4}\left[\frac{\rho_s}{\rho_{ds}}\left(d^2 - 2\,d\;tg\alpha\;dx\right) - \frac{d^2}{2\,\rho_{ds}}\,d\,\rho_s\right] =$$

$$= \frac{\pi}{4}\left[k\left(d^2 - 2\,d\;tg\alpha\;dx\right) + \frac{d^2}{2\,\rho_{ds}}\,d\,\rho_s\right]$$

- using the formula $k = 1 - \rho_s/\rho_{ds}$

Continuity equation for the conveying gas

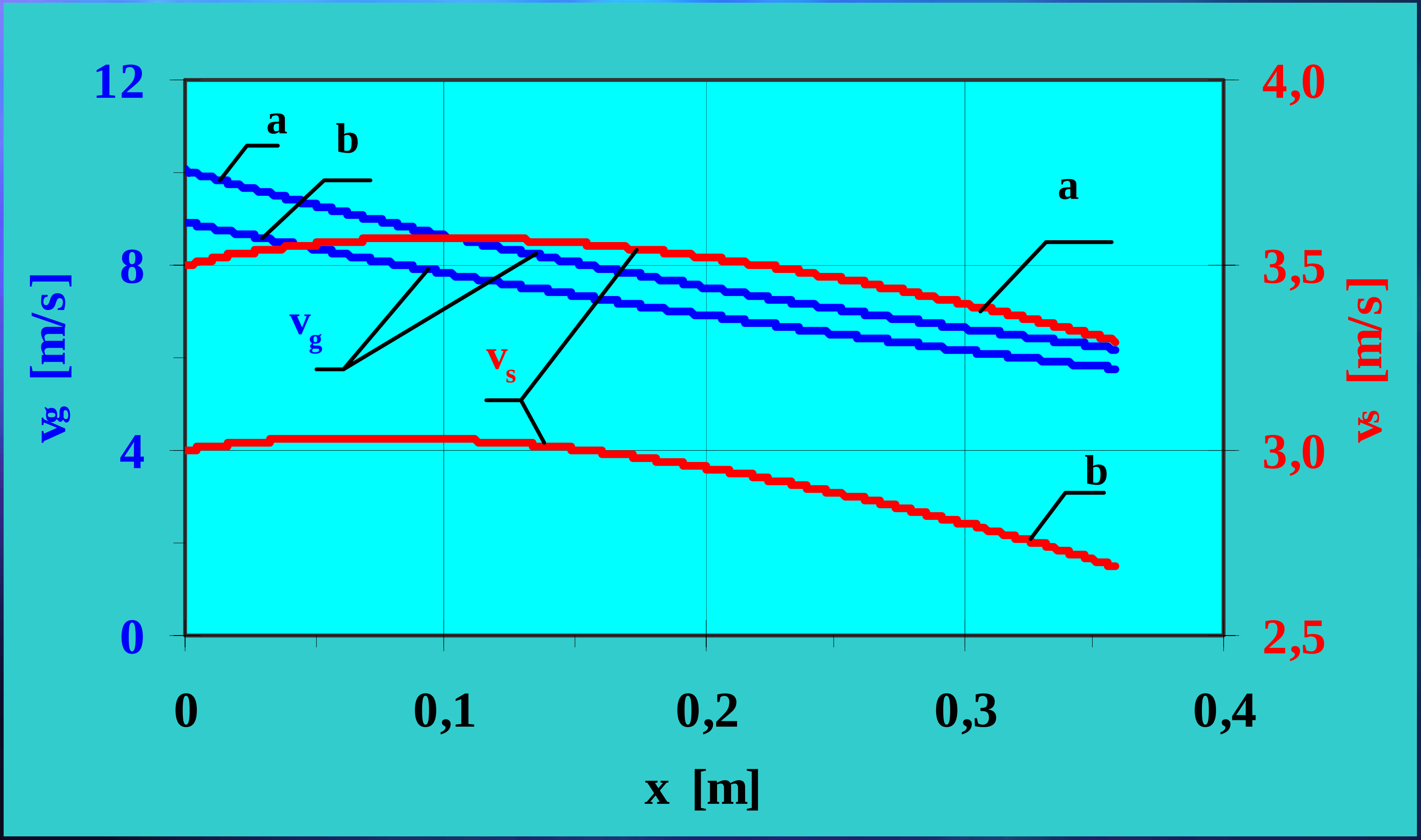
$$\frac{dp}{dx} = -\frac{p}{v_g}\frac{dv_g}{dx} + \frac{p}{k\,\rho_{ds}}\frac{d\,\rho_s}{dx} - \frac{p}{d}4\,tg\alpha$$

Momentum equation for the conveying gas

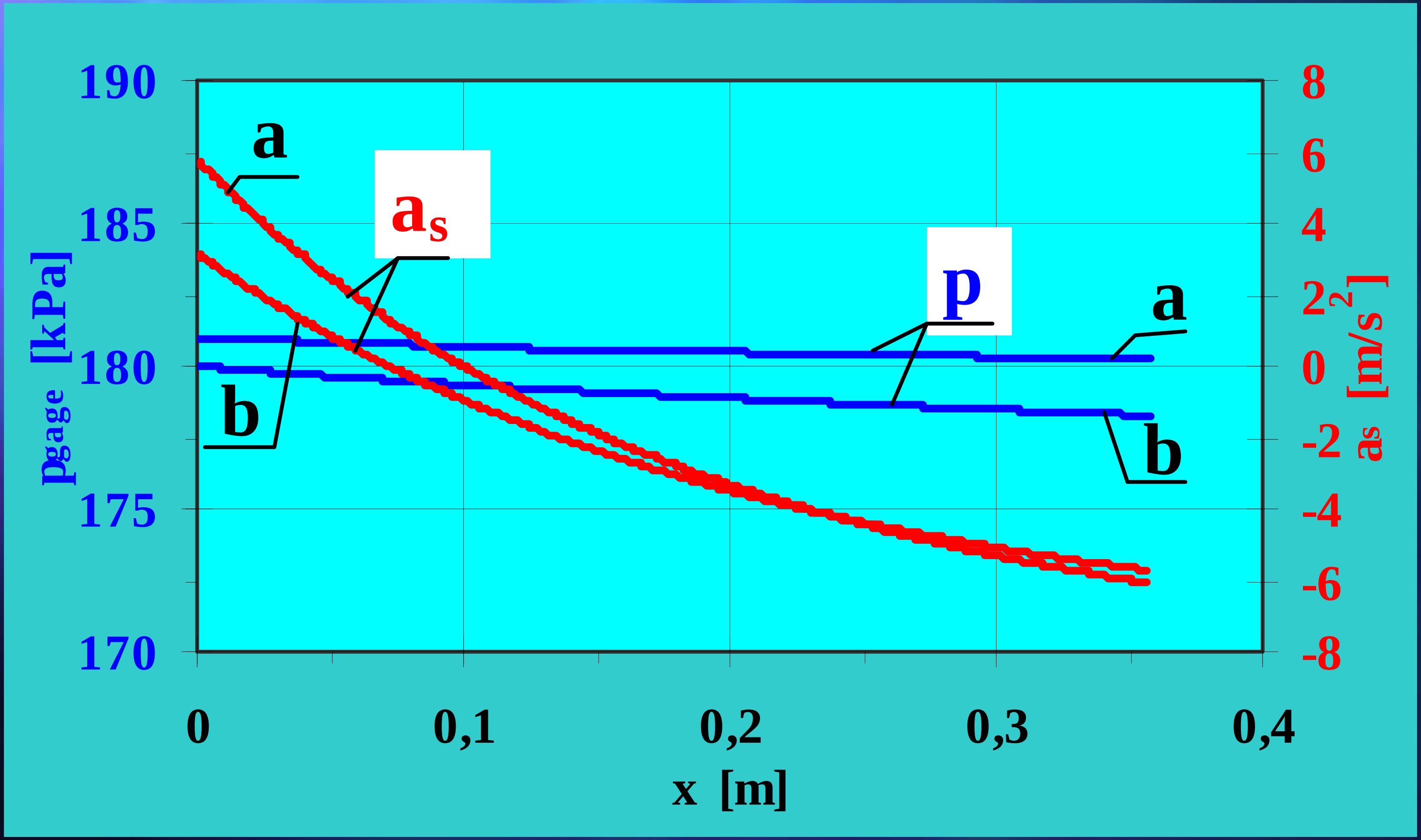
$$\frac{dv_g}{dx} = \frac{2\rho_{go} v_g^2}{\rho_{go} v_g^2 - 2p_o}\left[-\frac{p_o}{k\,\rho_{ds}\,\rho_{go}\,v_g}\frac{d\rho_s}{dx} + \frac{4 v_g\, tg\alpha}{d}\frac{2\rho_{go} v_g^2}{\rho_{go} v_g^2 - 2p_o} - \right.$$

$$\left. -\frac{\rho_s A_o C_D}{4k\, m_1 v_g}\left(v_g - v_s\right)^2 - \frac{\rho_s g\, p_o}{2k\,\rho_{go} v_g\, p}\left(\sin\beta + \mu\cos\beta\right)\right]$$

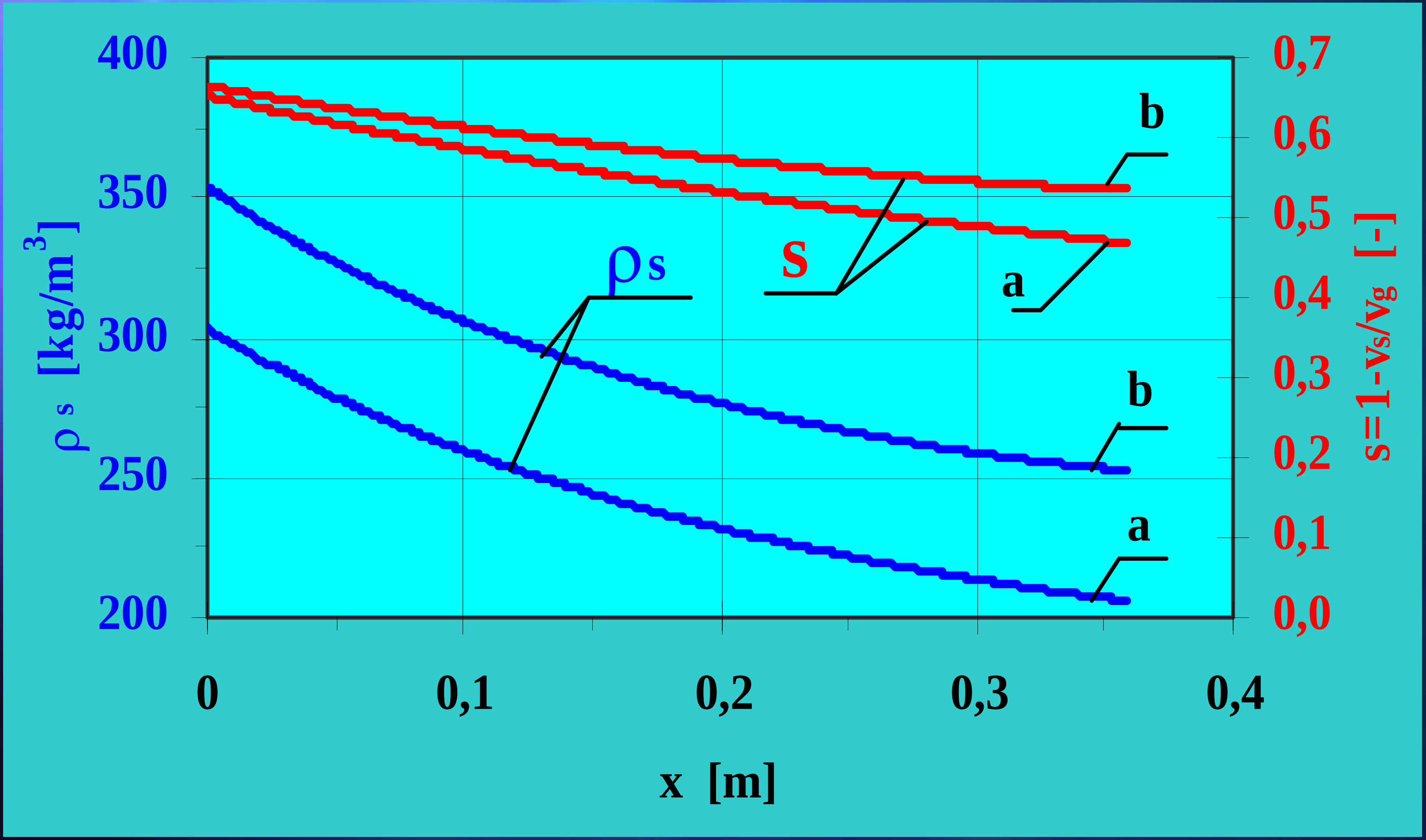
Gas- and solid material velocity in function of diffuser length



Pressure and material particle acceleration in function of diffuser length



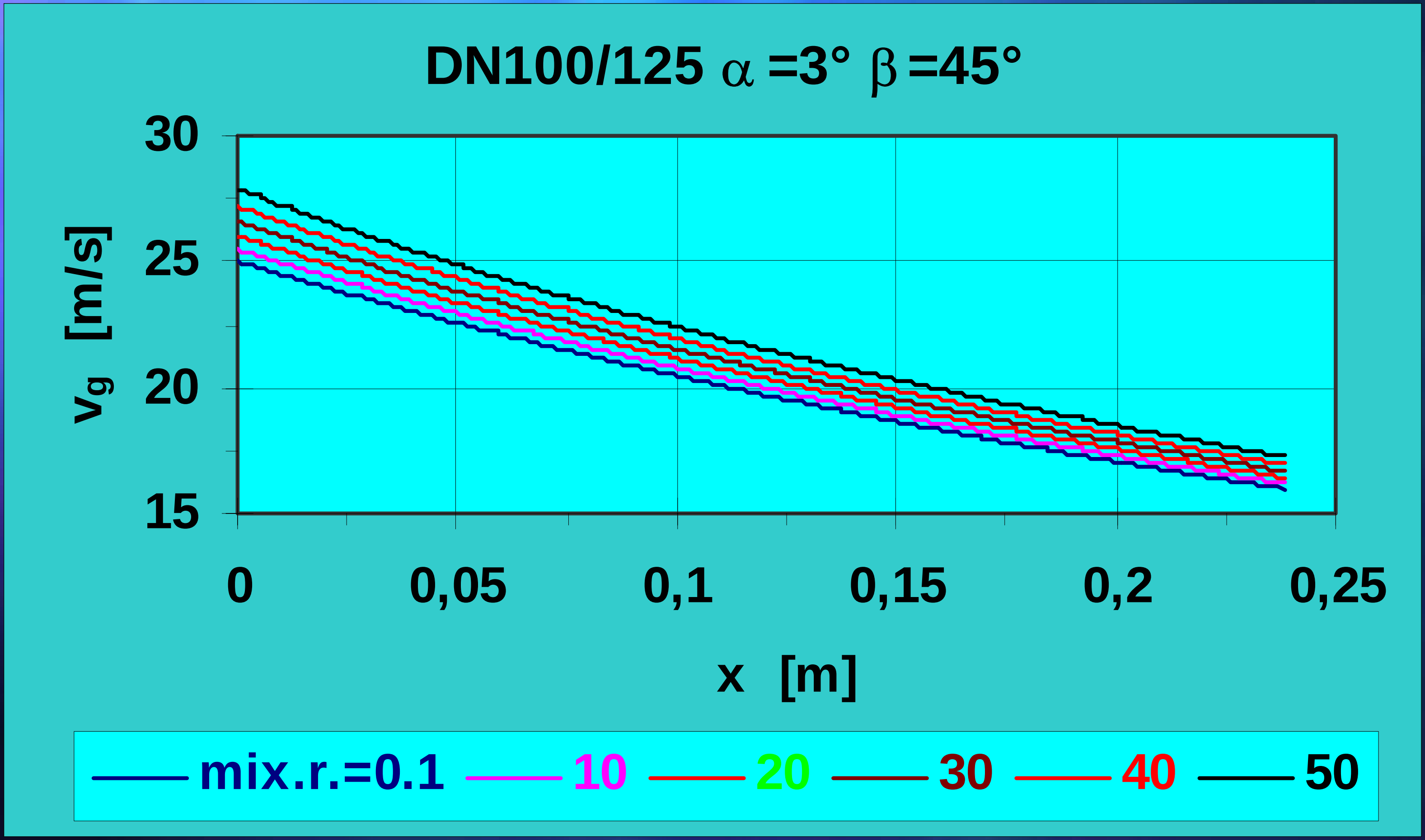
Particle concentration and slip in function of diffuser length



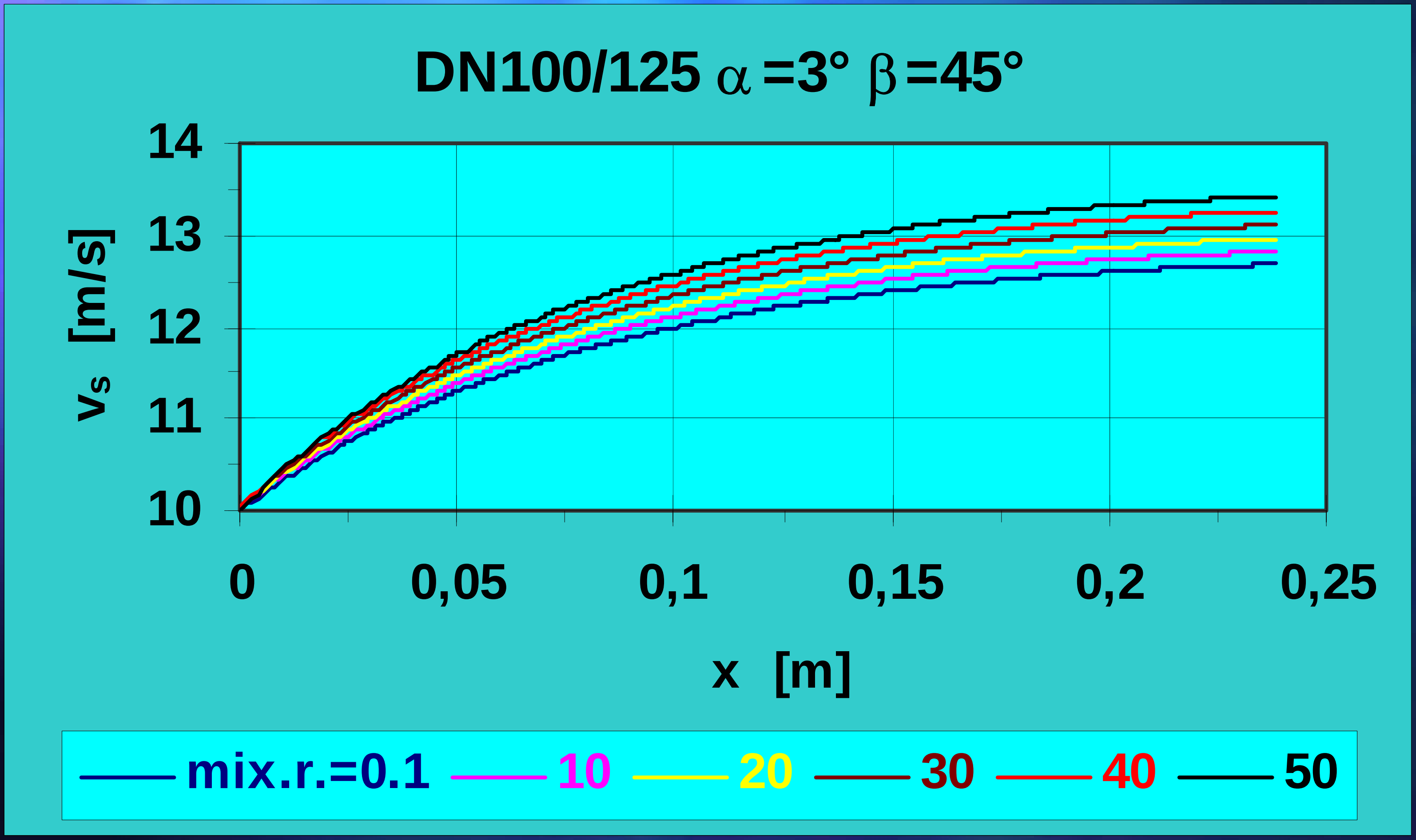
Examination of the effects of most important parameters

- Mixing ratio
- Inclination of pipe
- Slip
- Particle size
- Particle density
- Angle of diffuser
- Concentration

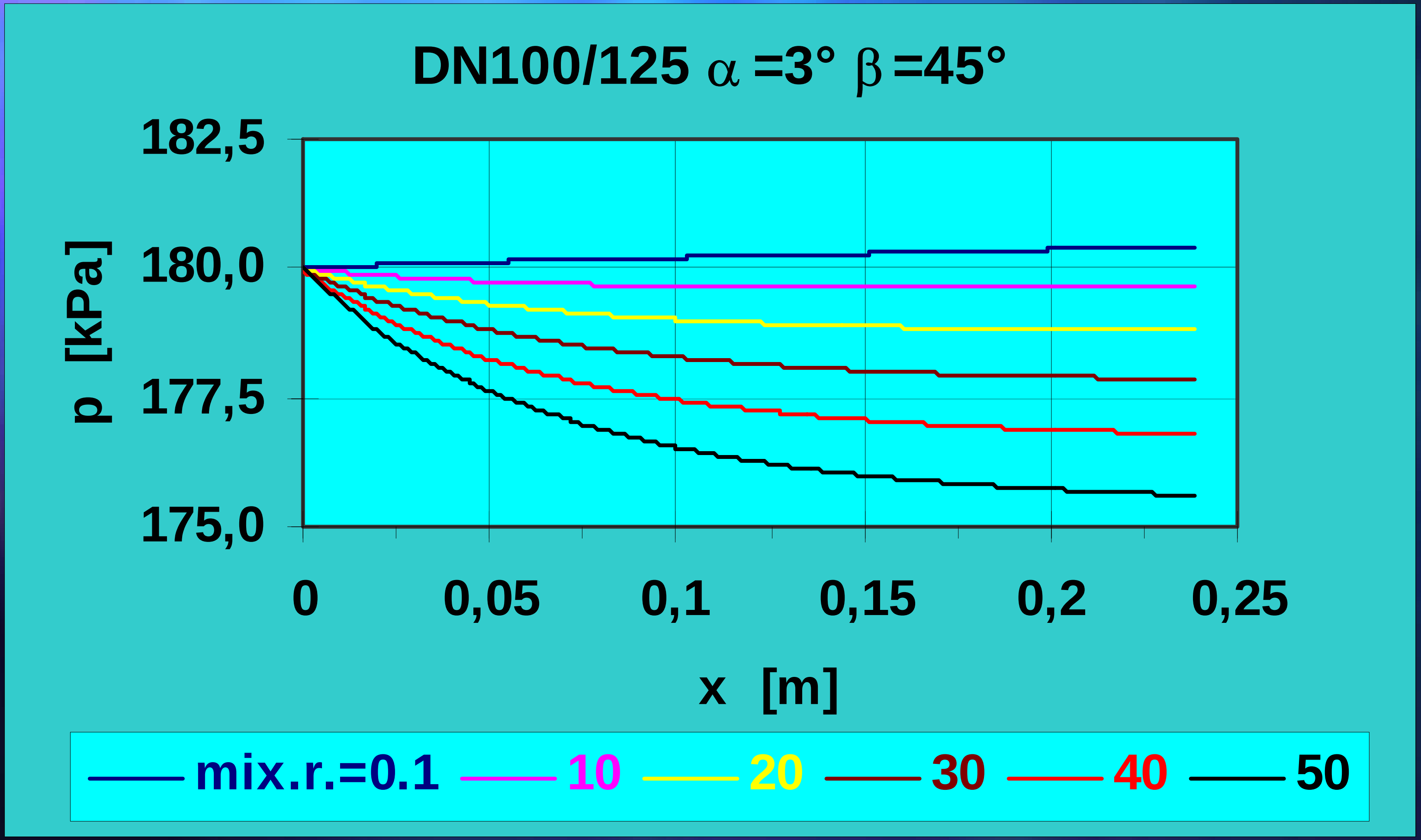
Parameter: mixing ratio



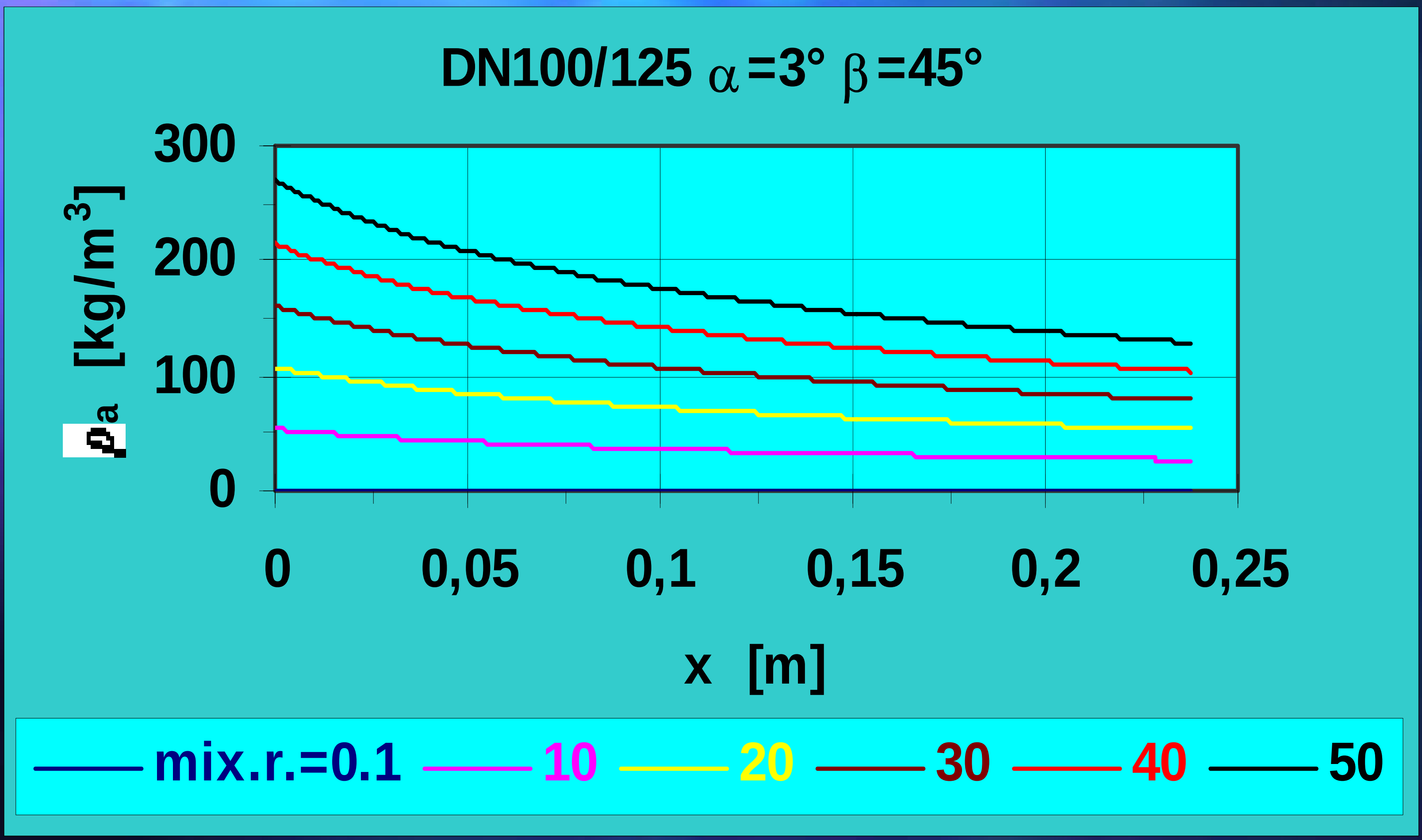
Parameter: mixing ratio



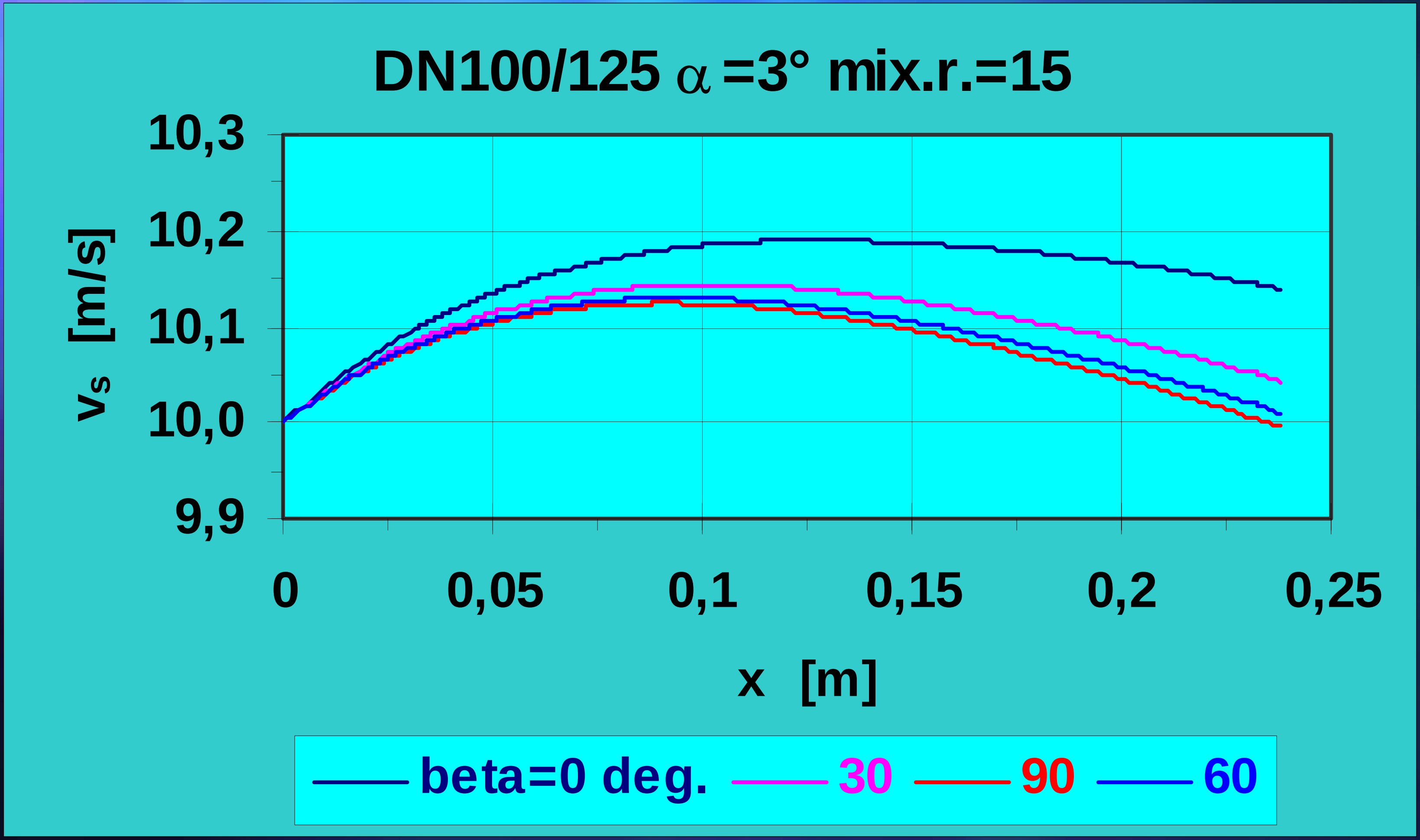
Parameter: mixing ratio



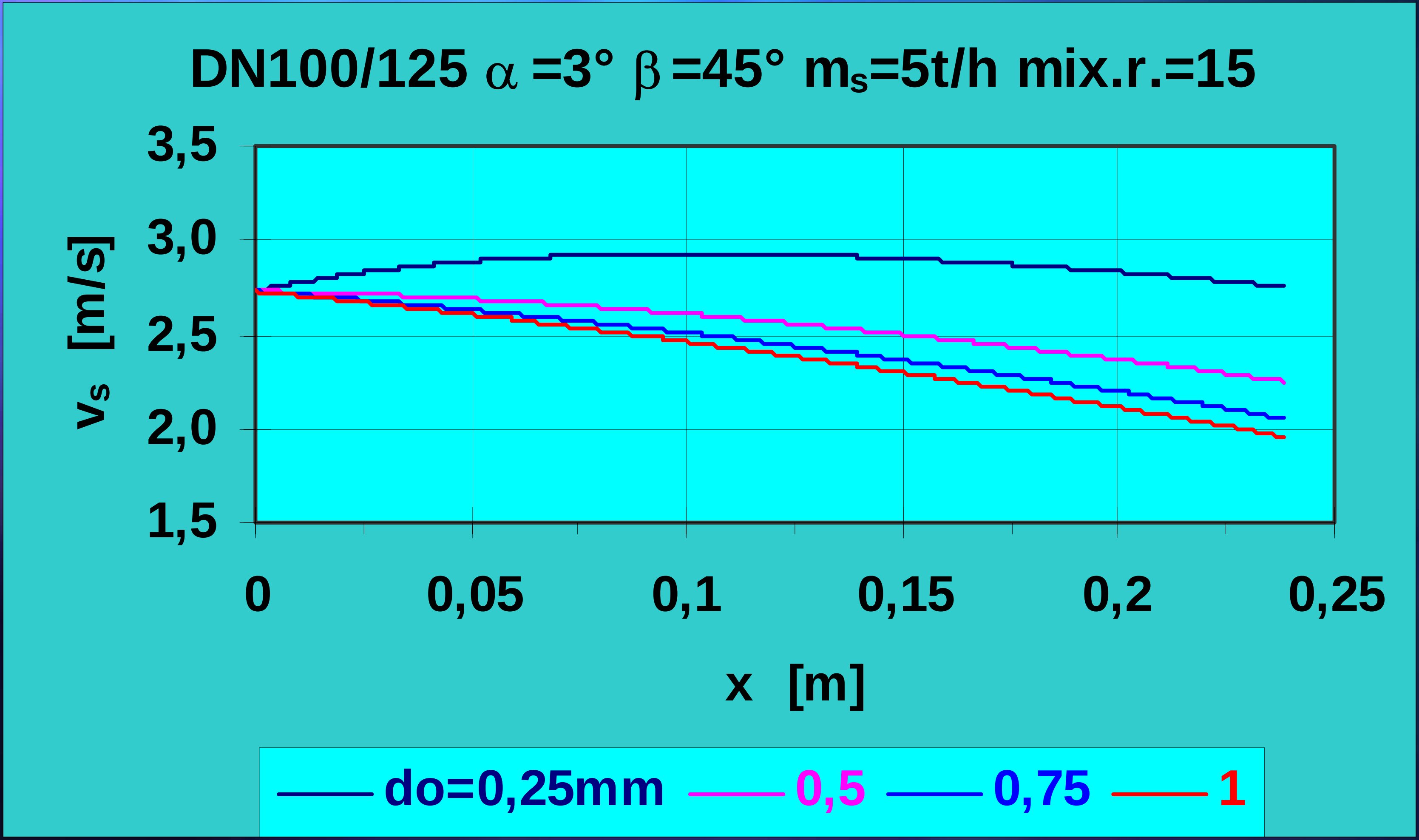
Parameter: mixing ratio



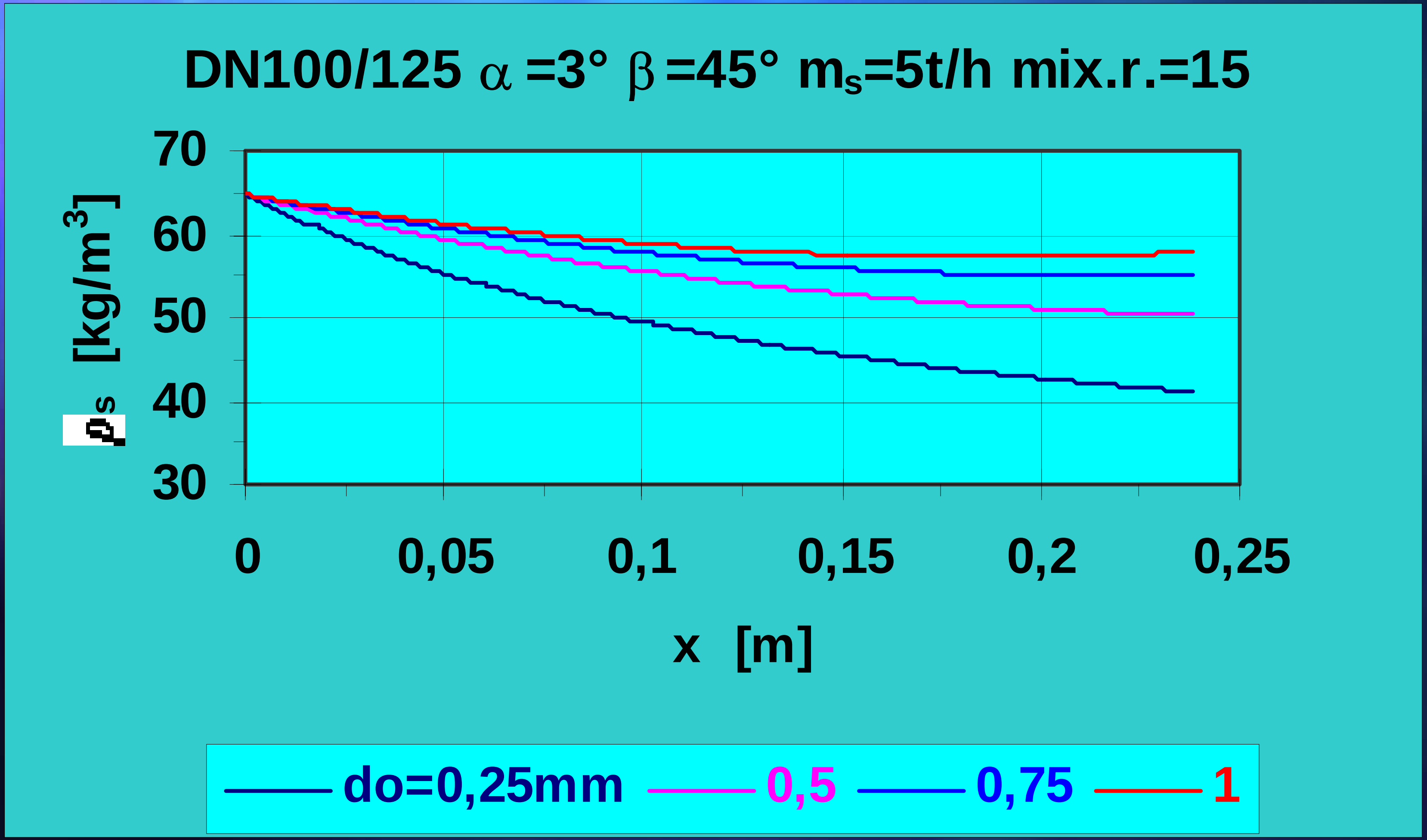
Parameter: inclination of pipe



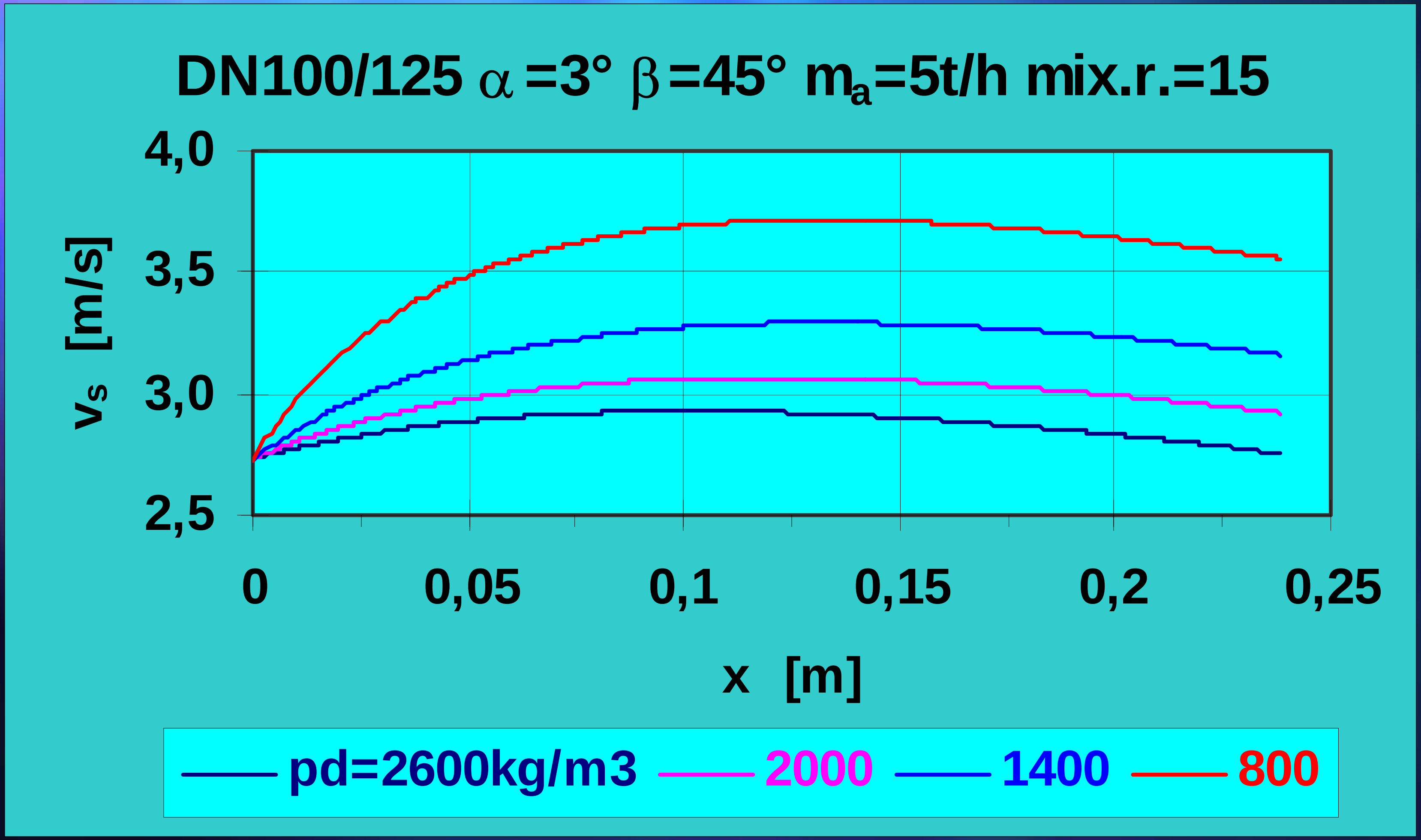
Parameter: diameter of the particle



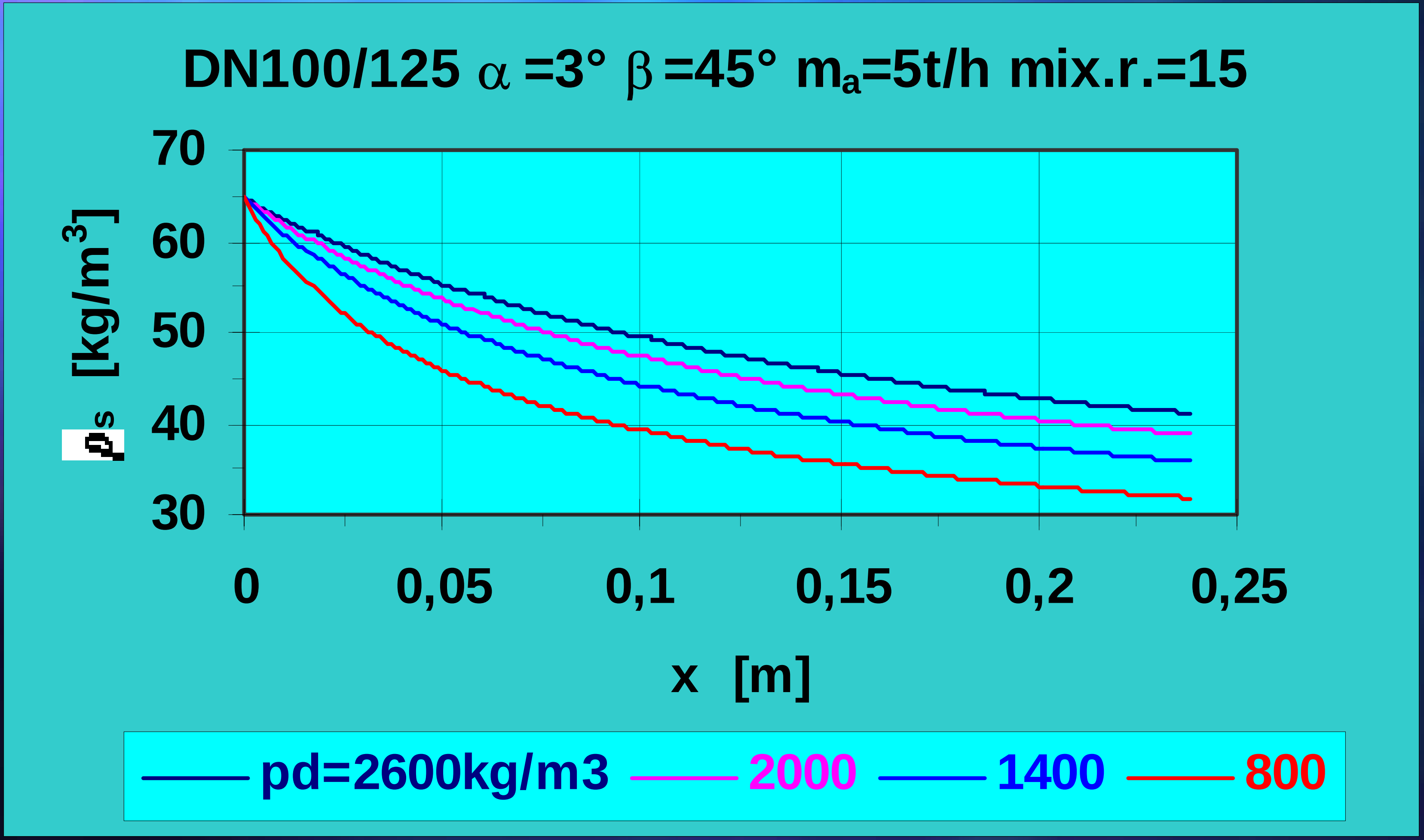
Parameter: diameter of the particle



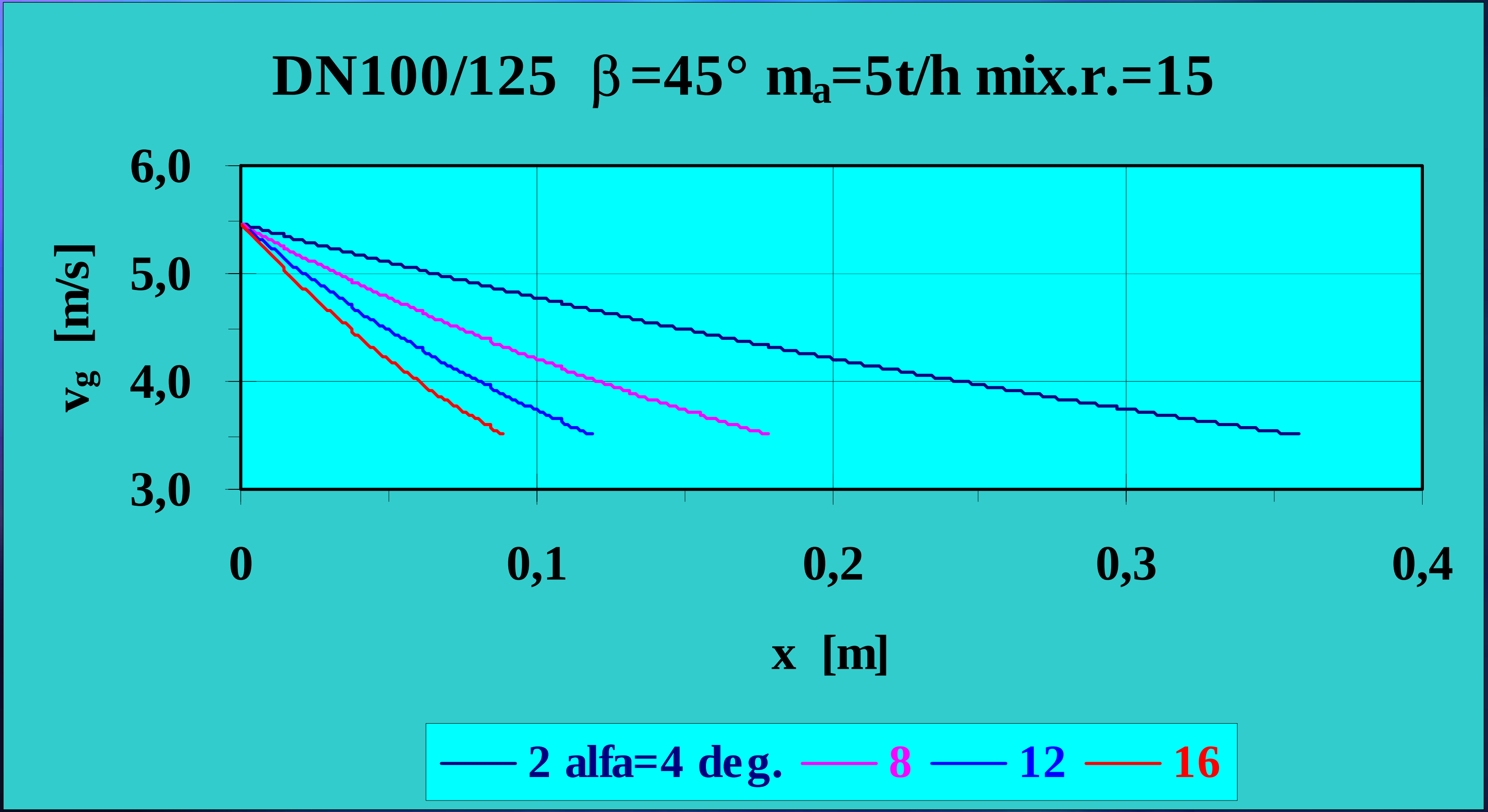
Parameter: particle density



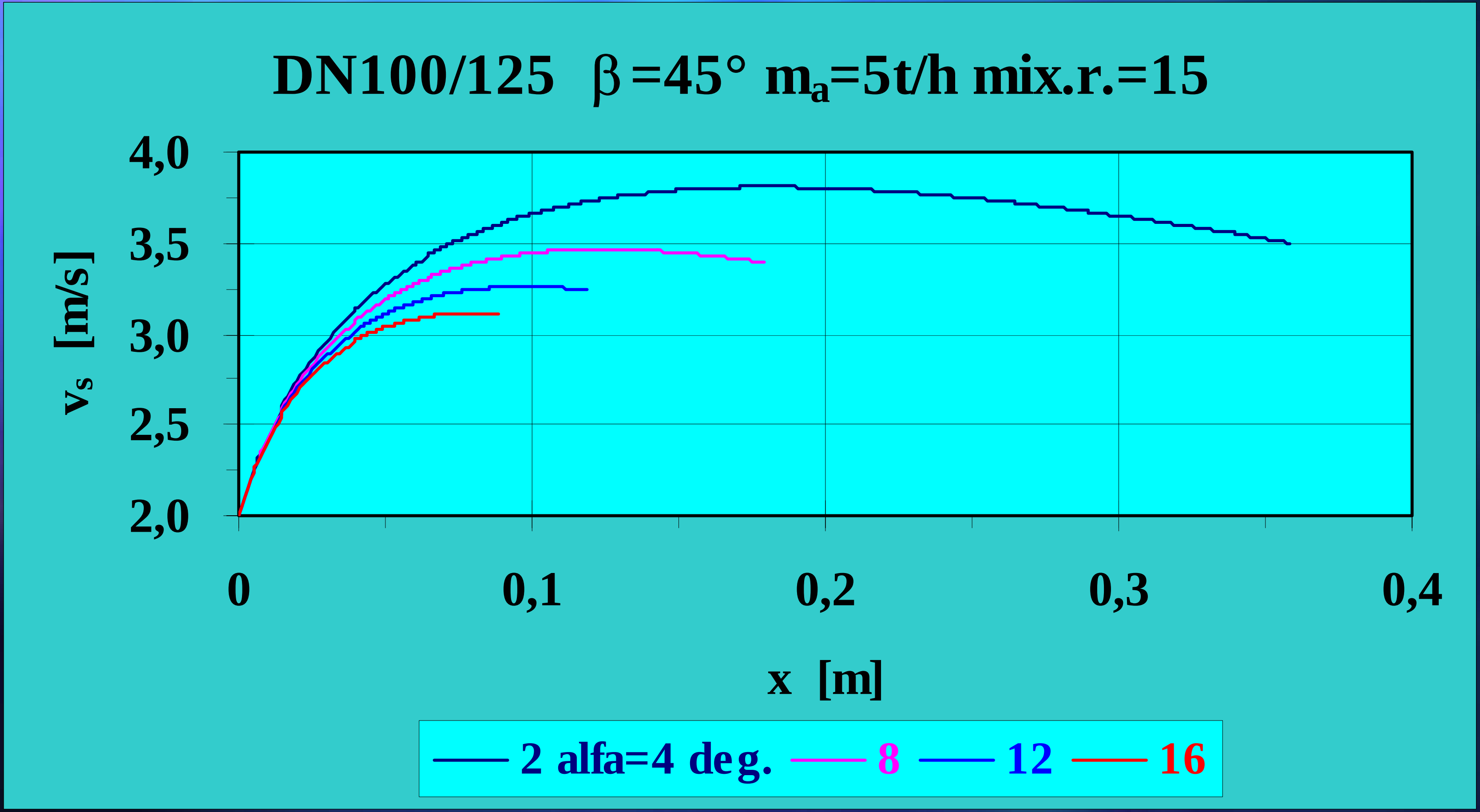
Parameter: particle density



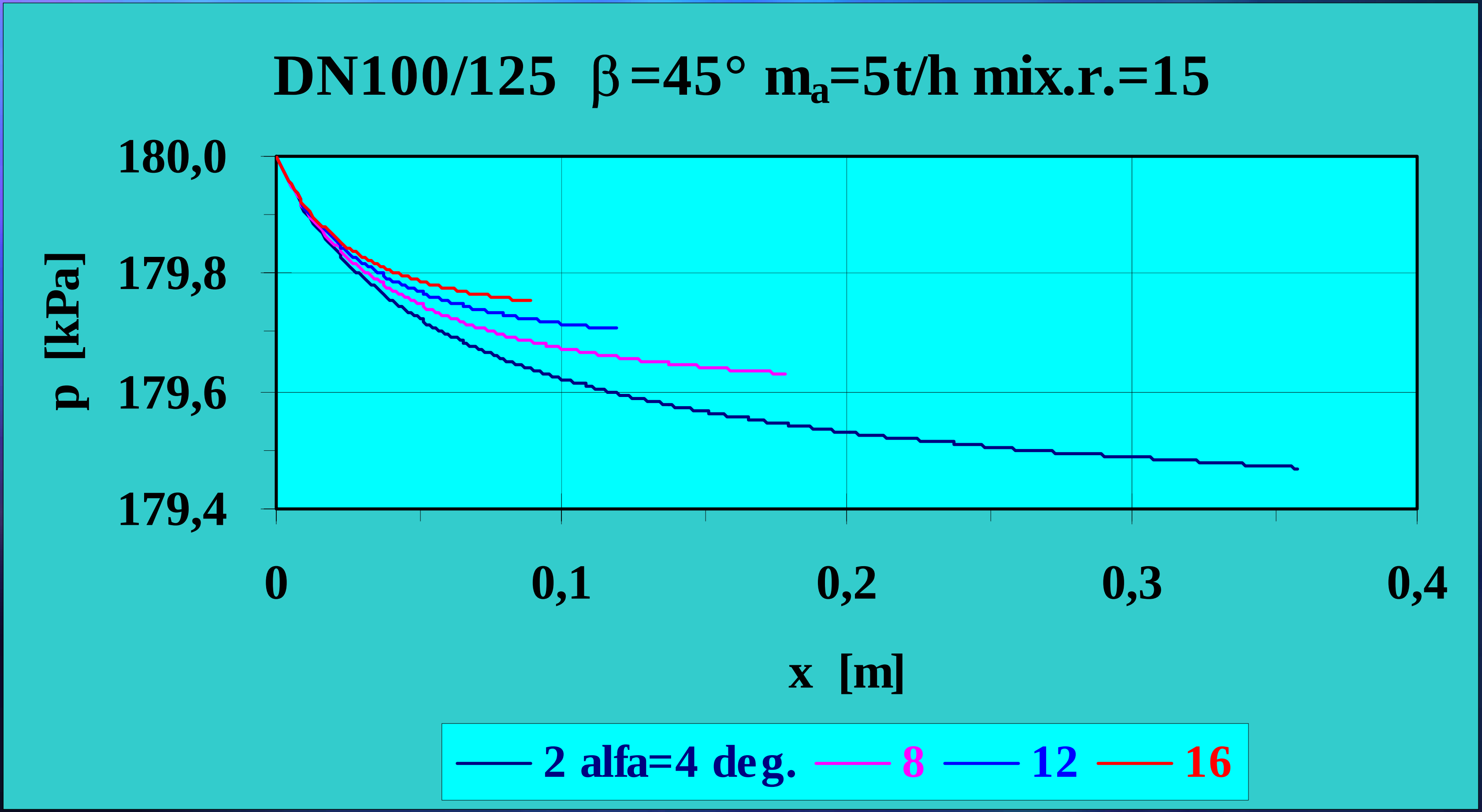
Parameter: diffuser angle



Parameter: diffuser angle



Parameter: diffuser angle



Parameter: diffuser angle