Chaotic oscillations of gas bubbles under dual-frequency acoustic excitation

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ABSTRACT

Chaotic oscillation of bubbles in liquids reduces the efficiency of the sonochemical system and should be suppressed in the practical applications. In the present paper, a chaos control method based on the dual-frequency approach is numerically investigated and is proved to be an effective method even for cases with intensive energy input. It was found that the chaos could be successfully suppressed by the application of dual-frequency approach in a wide range of parameter zone (even with high acoustic pressure amplitude). Furthermore, influences of power allocation between two waves on the chaos control are quantitatively discussed with clear descriptions of the routes from stable oscillations to chaos.

1. Introduction

Currently, acoustic cavitation is being widely employed in the sonochemistry including the promotions of the chemical reactions \cite{1,2}, the chemical synthesis \cite{3}, the surface cleaning \cite{4} and nanostructuring \cite{5}. In many applications of the cavitation effects, the chaotic oscillations of bubbles should be avoided because these bubbles will not oscillate following a controllable route, leading to the difficulties in the design of sonochemical reactors. Therefore, in order to avoid some undesirable results (e.g. efficiency reduction of the sonochemical reactor), control of the chaotic oscillations of the bubbles is an essential topic for the cavitation-enhanced sonochemical effects.

Among the chaos control strategies, dual-frequency technique is one of important chaos control method for the nonlinear system with following characteristics. Firstly, it is convenient for the implementation. In the dual-frequency chaos control method, the existing system does not need much modification except adding an extra acoustic wave into the system for the purpose of the suppress of the chaos. Secondly, the dual-frequency approach has been widely proved to be a highly effective method for many systems. In the literature, the nonlinear systems with successful chaos control include the electrical circuits, the duffing oscillator, the duffing-van der Pol oscillator, and the Frenkel-Kontorova chains. For reviews of dual-frequency chaos control technique, readers are referred to Nayfeh and Mook \cite{6}, Fradkov et al. \cite{7}, and Chacón \cite{8}. Thirdly, the dual-frequency approach is suitable for the bubble oscillator. For the oscillating bubbles in certain types of applications, many parameters of the system (e.g. the viscosity, the surface tension, and the speed of sound of surrounding liquids) are all fixed by the types of applications. Hence, as an external field, the dual-frequency approach could be independent of the types of applications, being a general method for many emerging fields. In the literature, the dual-frequency approach has been proved to be effective for promoting cavitation bubble effects \cite{9–21}. Furthermore, the bubble dynamics has been actively involved into the design of the sonochemical reactors \cite{19}. In the literature, many physical effects induced by the dual-frequency excitation have been intensively studied, including the mass transfer \cite{16,17}, the phase diagrams \cite{14}, the acoustical scattering \cite{18}, the secondary Bjerknes force between bubbles \cite{19}, the combination of power in the frequency-response curve together with the traditional main bands, harmonics and subharmonics. Furthermore, dual-frequency technique could be also applied to the modification of bubble distributions in the practical applications through changing the sign of forces between bubbles \cite{19}. For the nonlinearity (e.g. bifurcation), Behnia et al. \cite{12} briefly studied the response of a spherical bubble to a special case of dual-frequency technique (with equal amplitudes of two acoustic sources). However, the ability of the dual-frequency approach for suppressing the chaos has not been studied in great detail for the general purpose (e.g. within a large parameter zone).

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In the present paper, the dual-frequency chaos control method is numerically investigated within a wide range of parameter zone (e.g. the power allocation between two acoustic waves and acoustic amplitude). The sections of the present paper are organized as follows. Section 2 introduces the basic equations and numerical methods for studying the chaotic oscillations of bubbles. Section 3 discusses the methods for chaos identification. Section 4 proves the effectiveness of the dual-frequency chaos control method through several examples. Section 5 further reveals the examples of the route to chaos in the dual-frequency approach. Section 6 concludes the main findings of the present paper with remarks.

2. Equations and methods

In this section, the equations and method employed for the prediction of the chaotic oscillations of gas bubbles in liquids will be introduced. The equation of bubble interface motion adopted here is the Keller-Miksis equation [22],

\[
\frac{1}{2} \left( \frac{R}{c_i} \right)^2 \frac{d}{dt} \left( \frac{R}{c_i} \right)^2 = \left( 1 + \frac{R}{c_i} \right) \frac{p_{eq}(R,t) - p_\text{t}(t)}{\rho_l} + \frac{R}{\rho_l c_i} \frac{d}{dt} \left[ \frac{p_{eq}(R,t) - p_\text{t}(t)}{\rho_l c_i} \right],
\]

where

\[
p_{eq}(R,t) = \left( \frac{p_0 + 2\sigma}{R_0} \right)^3 - \frac{4\mu R}{R - 2\sigma},
\]

\[
p_t(t) = p_0 \left[ 1 + \eta \cos(\omega t_1 + \varphi) + \eta \cos(\omega t_2 + \varphi) \right].
\]

Here \( R \) is the instantaneous bubble radius; the overdot denotes the time derivative; \( c_i \) is the speed of sound in the liquid; \( \rho_l \) is the density of the liquid; \( t \) is the time; \( P_0 \) is the ambient pressure; \( \sigma \) is the surface tension coefficient; \( R_0 \) is the equilibrium bubble radius; \( \kappa \) is the polytropic exponent; \( \mu \) is the viscosity of the liquid; \( \eta_1 \) and \( \eta_2 \) are the non-dimensional amplitudes of the external acoustic excitation; \( \omega_1 \) and \( \omega_2 \) are the angular frequencies of the two external acoustic waves (assuming that \( \omega_1 < \omega_2 \) for convenience); \( \varphi \) is the relative phase between the two external acoustic waves. For convenience, we set \( \varphi = 0 \) here. Also, the energy dissipation through the thermal effects was ignored [23,24]. Eqs. (1)–(3) is an ordinary differential equation and could be directly solved by using the fourth order Runge-Kutta method.

For convenience of discussions, the non-dimensional parameters are employed as listed as below

\[
X_{\max} = (R_{\max} - R_0)/R_0;
\]

\[
\tilde{\alpha}_1 = \frac{\alpha_1}{\alpha_0}; \tilde{\alpha}_2 = \frac{\alpha_2}{\alpha_0}
\]

\[
\tilde{P}_e = \frac{P_e}{P_0};
\]

\[
N = \frac{\tilde{c}_1}{\tilde{c}_2};
\]

\[
\alpha_0^2 = \frac{1}{\rho_l R_0^2} \left[ 3\kappa \left( \frac{P_0 + 2\sigma}{R_0} \right) \frac{2\sigma}{R_0} \right],
\]

\[
P_e = P_0 \tilde{c}_1^2 \left( 1 + N^2 \right).
\]

Here \( R_{\max} \) is the local maximum of the bubble radius during oscillations; \( \alpha_0 \) is the linear natural frequency of the oscillating bubbles; \( P_e \) is the overall acoustic amplitude of the dual-frequency excitation. The following constants are employed in calculations: \( P_0 = 1.013 \times 10^5 \text{ Pa}; \) \( \rho_0 = 998.20 \text{ kg/m}^3; \) \( \mu = 1.0 \text{ mPa·s}; \) \( \sigma = 0.0728 \text{ N/m}; \) \( \kappa = 1.85 \text{ m/s}; \) \( \tilde{c} = 1486 \text{ m/s}; \) \( \kappa = 1.33 \).

3. Method for chaos identification

In this section, the method employed in the present paper for the identification of chaos will be introduced. Firstly, cases with single-frequency excitation will be investigated. For single-frequency excitation, the frequency of acoustic wave is represented simply by \( \omega \). According to Eqs. (5) and (8), the frequency could be non-dimensionalized as \( \tilde{\omega} = \omega/\omega_0 \). The overall acoustic amplitude of the single-frequency excitation is still denoted as \( P_e \). Fig. 1 shows the variations of the bubble interface motion speed (\( dR/dt \)) versus the non-dimensional bubble radius (\( R/R_0 \)) under single-frequency excitation without chaos. \( \tilde{\omega} = 0.35 \). \( P_e/P_0 = 0.05 \) is employed. As shown in Fig. 1, the bubble oscillates periodically with a fixed pattern. However, when the amplitude increases, the bubble behaviour will be quite different. Fig. 2 shows the oscillations of the same bubble in Fig. 1 but with larger amplitude (\( P_e/P_0 = 0.85 \)). As shown...
in Fig. 2, the oscillations of bubble become chaotic (i.e. the random patterns of bubble oscillations). The curves shown in Figs. 1 and 2 were also named as the basin of the attractor in the literature. This technique for the chaos identification has been also employed by many researchers (e.g. Fig. 2 of Moholkar et al. [25]).

Now, the cases with dual-frequency excitation will be discussed. Figs. 3 and 4 show the variations of the bubble interface motion speed \( \frac{dR}{dt} \) versus the non-dimensional bubble radius \( \frac{R}{R_0} \) under dual-frequency excitation without and with chaos respectively. The same parameters were employed in those figures except \( N \) (\( N = 2.0 \) and \( N = 1.4 \) respectively). Different with the case under single-frequency excitation, the chaos could be generated by adjusting the parameter relating with the energy allocation between two acoustic waves (i.e. \( N = 2.0 \) without chaos but \( N = 1.4 \) with chaos).

The above identification method is effective for studying several demonstrating examples but is not practically achievable when a large parameter zone is involved. For example, for dual-frequency cases, the parameter zone include many parameters (e.g. \( \Omega_1, \Omega_2, P_1/P_0, N \)). Hence, based on the above understanding of the chaos during bubble oscillations, a more effective method for the chaos identification with many parameters involved will be shown and discussed. Figs. 5 and 6 shows the variations of the non-dimensional change of bubble radius \( \left[X = \frac{(R - R_0)}{R_0}\right] \) during its oscillation under dual-frequency excitation versus time without and with chaos respectively. The solid black dots demonstrate the local maximum bubble radii during the oscillations. The parameters are the same as those in Fig. 4, \( \Omega_1 = 0.35, \Omega_2 = 0.70, P_1/P_0 = 0.639 \).
sented by \( P \) could be expressed as the following condition \([12,26]\):

\[
P \equiv \max \{ (R, \dot{R}): \dot{R} = 0 \}.
\]

For a detailed definition of bifurcation diagram, readers are referred to Lauterborn and Parlitz \([27]\). Therefore, in the present paper, the local maximum bubble radii during bubble oscillations (represented by \( X_{\text{max}} \) in terms of non-dimensional value) are used to obtain the bifurcation diagrams. For each certain set of parameters, Eqs. (1)–(3) are solved for a long period in order to obtain the bifurcation diagrams. In order to eliminate the initial transient behaviour, only the final oscillations (with about 100 cycles) are employed.

4. Effectiveness of the dual-frequency approach

In this section, the effectiveness of the dual-frequency approach on the suppression of the chaos will be demonstrated. The bifurcation diagrams are constructed to quantitatively show the status of bubble oscillations. A large parameter zone (consisting of \( P_e/P_0 \) and \( N \)) is investigated for the completeness. In this section, if not specify, \( \tilde{n}_1 = 0.35 \) is selected as the frequency for the single-frequency approach. For the dual-frequency cases, its harmonics \( \tilde{n}_2 = 2\tilde{n}_1 = 0.70 \) is added to suppress the chaos.

Fig. 7 shows bifurcation diagram for single-frequency approach with \( \tilde{n}_1 = 0.35 \). For single-frequency case, chaotic oscillations (e.g. regions with a large amount of dots in the figures) can be found when \( P_e/P_0 \) is within the range of \([0.80–0.95]\) and \([1.2,2.1]\). For other regions, no chaos could be observed. Figs. 8 and 9 show the bifurcation diagrams for the dual-frequency approach with \( N = 1.0 \) and \( N = 2.0 \) respectively. When \( N = 1.0 \), parts of the chaotic region have been suppressed. For example, no chaotic oscillation is shown in regions with \( P_e/P_0 = [0.7,1.3] \) in Fig. 8 but those regions suffer from the chaos (as shown in Fig. 7). Comparing with Fig. 7, this is a very promising results noticing that parts of chaotic regions in Fig. 7 have been successfully suppressed by the dual-frequency approach. If we further adjust the power allocation with \( N = 2.0 \), the results are better noticing that nearly no chaotic oscillation can be observed with \( P_e/P_0 < 2.0 \). Hence, one can find that the power allocation (represented by \( N \)) is important for the performance of the dual-frequency approach.

Fig. 7. Bifurcation diagram of \( X_{\text{max}} \) varying with \( P_e/P_0 \) for the single-frequency approach with \( \tilde{n}_1 = 0.35 \).

Fig. 8. Bifurcation diagram of \( X_{\text{max}} \) varying with \( P_e/P_0 \) for the dual-frequency approach with \( N = 1.0 \).

Fig. 9. Bifurcation diagram of \( X_{\text{max}} \) varying with \( P_e/P_0 \) for the dual-frequency approach with \( N = 2.0 \).

Fig. 10. Bifurcation diagram of \( X_{\text{max}} \) varying with \( N \) for the dual-frequency approach with \( P_e/P_0 = 1.4 \).
Now, the influences of power allocation (referring to \(N\)) are discussed quantitatively with the aid of bifurcation diagram. Fig. 10 further shows the bifurcation diagram for the dual-frequency cases with \(P_e/P_0 = 1.4\) and \(N\) varying between 0.5 and 20. For the single-frequency case, readers are referred to the red line marked in Fig. 7. As shown in Fig. 10, when \(N > 1.0\), the chaotic oscillations observed in the Fig. 7 could be successfully suppressed. We also noticed that for \(N < 1.0\), there are still significant chaos. Hence, it is quite essential to understand the route to chaos in the dual-frequency approach in great details.

5. Route to chaos under the dual-frequency approach

In this section, two examples for the route to chaos in the dual-frequency approach are given with discussions. Fig. 11 shows the first example of the route to chaos with \(P_e/P_0\) varying between 0.600 and 0.639. It should be emphasized that the bifurcation and chaos are both very sensitive to the parameter \(P_e/P_0\) hence a very subtle change of \(P_e/P_0\) was given during our calculations. For \(P_e/P_0 = 0.600\) (Fig. 11a), there is no bifurcation and chaos observed. For \(P_e/P_0 = 0.628\) (Fig. 11b), the bifurcation (e.g. splitting into two lines) happens. For \(P_e/P_0 = 0.630\) (Fig. 11c), \(0.633\) (Fig. 11d) and \(0.635\) (Fig. 11e), the regions covered by the bifurcation gradually increase. For \(P_e/P_0 = 0.639\) (Fig. 11f), the chaos is finally shown.

Fig. 12 further shows another route to chaos under the dual-frequency approach with \(P_e/P_0\) varying between 1.000 and 1.120. For \(P_e/P_0 = 1.000\) (Fig. 12a), there is no bifurcation and chaos shown in the figure. For \(P_e/P_0 = 1.050\) (Fig. 12b) and 1.100 (Fig. 12c), the bifurcation (e.g. line splits) happens in the two branches. For \(P_e/P_0 = 1.112\) (Fig. 12d), more bifurcation is shown with line split into four. For \(P_e/P_0 = 1.114\) (Fig. 12e), intensive bifurcation is shown in all the branches. For \(P_e/P_0 = 1.120\) (Fig. 12f), the final chaos is demonstrated with clear signals shown in all the branches. The understanding of the above route to chaos is very helpful for the design of the dual-frequency system for suppressing the chaos in the regions of interest.

6. Conclusion

In the present paper, the effectiveness of the dual-frequency approach for suppressing the chaotic oscillations of gas bubbles has
been demonstrated through the numerical simulations. The paramount parameters relating with the chaos were all discussed in the present paper. To aid the analysis, the chaos was identified with the bifurcation diagram constructed using the local maximum bubble radius during the bubble oscillations. Results of the single-frequency excitation were also given for the convenience of comparisons.

As shown in our results, the dual-frequency approach could successfully remove the chaos out of regions of interest in many cases. Even for a very high acoustic pressure amplitude, a chaos-free region could still be accomplished with choosing appropriate parameters (e.g. the power allocations between two acoustic waves). The route from stable oscillations to the chaos in the dual-frequency approach is also demonstrated with the aid of two examples for the further design of the system. In the future work, the chaotic oscillations will be further explored with the parameters of the sonochemical reactors in the engineering practice.

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Y. Zhang, Y. Zhang


