Flow around rectangular cylinders: Pressure forces and wake frequencies

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Abstract

This paper concerns an experimental investigation of the flow around and pressure forces on fixed (non-vibrating) rectangular cylinders at angles of attack $0^\circ - 90^\circ$. Pressure forces and moments for cylinders with side ratios $B/A = 1, 1.62, 2.5$ and $3$ (shortest side $A = 20$ mm) were estimated from measurements of static pressure distributions at mid-span. Wake frequencies and associated Strouhal numbers were determined from hot wire measurements in the near-wake regions ($A = 4$ and $20$ mm). With the smaller cylinders 12 side ratios within $B/A = 1 - 5$ were investigated. The free stream turbulence intensity was less than 0.06%, blockages less than 5% and aspect ratios $L/A$ greater than 50. Reynolds numbers, based on $A$, ranged from about $Re = 400$ to $Re = 3 \times 10^4$ (pressure measurements from about $Re = 3 \times 10^3$). For the square cylinder, the measured pressure forces were used for calculations of quasi-steady galloping response in the plunging mode.

1. INTRODUCTION

In recent years, the problem of flow-induced vibrations of slender structures has become increasingly important. From an engineering point of view, the prediction whether a structure has a potential to experience damaging vibrations or not is very important. In addition, the study of bluff body wakes and its associated fluid forces and wake frequencies is of fundamental interest. The main objective of the paper is to present accurate and reliable data on the combined effects of Reynolds number and angle of attack (flow incidence) for the flow around and pressure forces on rectangular cylinders at various side ratios.

The flow configuration of the present investigation is depicted in Figure 1a. A fixed, large-aspect-ratio, rectangular cylinder with a side ratio $B/A$ is exposed to a low-turbulence cross flow with a constant free stream velocity $U_o$ which is at an angle $\alpha$ referred to the longer side of the cylinder (dimension $B$). The projected width $D$ in relation to the shorter side (dimension $A$), for angles of attack $\alpha = 0^\circ - 90^\circ$, is $D/A = \cos \alpha + (B/A) \sin \alpha$. Normally the Reynolds number was based on $A$, i.e. $Re = U_o A / \nu$. In some cases, however, it was based on $D$, i.e. $Re_D = Re(D/A)$. The Strouhal number was defined as $St = f_S D / U_o$, where $f_S$ is the peak frequency in the calculated power spectral density. The drag, lift and moment coefficients were also based on $D$ defining the drag, lift and pitching moment.
per unit length: $F_d = C_d D p U_o^2 / 2$, $F_l = C_l D p U_o^2 / 2$ and $M_e = C_M D^2 p U_o^2 / 2$, respectively.

Figure 1. (a) Fixed rectangular cylinder with side ratio $B/A$ at an angle of attack $\alpha$. (b) Rectangular cylinder at an angle of attack $\alpha_o$ at an instant where the cylinder is moving downwards with a velocity $-\dot{y}$.

A large amount of data has been gathered at angles of attack $\alpha = 0^\circ, 90^\circ$, see e.g. [1, 2, 3]. In those investigations, the side ratio usually is defined as the ratio between the side aligned with the flow and the side normal to the flow, $H/D$. However, except for the square section, see e.g. [4], the effects of flow incidence have not been extensively covered. Nevertheless, effects of flow incidence for $B/A \neq 1$, can be found in [5, 6, 7, 8, 9]. For a more complete review, the reader is referred to Knisely [9].

The present study comprises side ratios $B/A = 1 - 5$ at Reynolds numbers from about $Re = 400$ to $Re = 3 \times 10^4$ ($B/A = 1 - 3$, $Re > 3 \times 10^3$ for pressure forces and moments). When comparisons were possible, the present data agreed well with existing results.

Consider now the cylinder in Figure 1b which is at an angle of attack $\alpha_o$ relative to $U_o$, and which at that particular moment is moving downwards with a velocity $-\dot{y}$. As judged from the cylinder the effective free stream velocity then is at an angle $\varphi = \arctan(\dot{y} / U_o)$ from the actual free stream velocity. The aerodynamic downward force per unit length can be written as $F_Y = C_{FY} D_o p U_o^2 / 2$, where $C_{FY}$ is the transverse force coefficient and $D_o$ is the projected width at $\alpha_o$. In the quasi-steady theory of transverse galloping in the plunging mode, developed by Parkinson et al. ([10, 11]) with refinements by e.g. Novak [12], it is assumed that the instantaneous aerodynamic force on the moving cylinder is the same as would be measured with a fixed cylinder at an angle of attack $\alpha_o + \varphi$ and with a free stream velocity $U_R = \sqrt{U_o^2 + \dot{y}^2}$. The basic assumption for the quasi-steady analysis to be applicable is that the dominant vortex shedding frequency $f_s$ is much higher than the cylinder natural frequency of transverse oscillation $f_n$, i.e. at reduced
velocities $U_r = U_o/(2\pi f_0 D) \gg (2\pi St)^{-1} \sim 1$. According to the theory, the cylinder is stable or unstable at rest at $\alpha_0$ depending on whether the slope of $C_f$ vs $\alpha$ at $\alpha_0 = A_1$ is positive or negative. When positive the initial galloping reduced velocity is equal to $\beta/(nA_1)$, where $\beta$ is the damping coefficient and $n$ is the mass parameter, respectively [12]. For further details and more sophisticated models including resonant vibrations the reader is referred to [13, 14, 15]. In the present study the quasi-steady approach was used to calculate the limiting galloping response for the square cylinder at $\alpha_o = 0^\circ$.

2. EXPERIMENTAL

2.1. Wind Tunnel and Models

The measurements were carried out in a low-speed closed-circuit wind tunnel with a working section of height 1.25 m, width 1.80 m and length 2.90 m. The free stream turbulence intensity was less than 0.06%. The cylinders were supported between two vertical plates which were grounded to the floor and mutually stabilized above the roof. The actual position for $\alpha = 0^\circ$ was found from the symmetry in the mean pressure distributions. The deviation from the horizontal was found to be of the order 0.3°. The accuracy in the angle of attack $\alpha$ was estimated to be better than $\pm 0.1^\circ$.

The free stream velocity $U_o$ was measured with a Pitot-static tube positioned 0.7 m upstream of the cylinders. The uncertainty in $U_o$ was estimated to be 0.6% at velocities higher than 3 m/s and about 1.5% at the lowest velocity $\sim 1.5$ m/s (constant odds 20:1).

Two sets of cylinder models were used. In the first set the shortest sides were about 3.9 mm (referred to as $A = 4$ mm) and in the other the shortest sides were about 19.5 mm (referred to as $A = 20$ mm). The models were all made of aluminium and were milled, grinded and finally polished in order to achieve (i) constant cross sections over the lengths, (ii) sharp edges and (iii) smooth surfaces. For the set with $A = 4$ mm the following 12 side ratios were investigated: $B/A = 1.00, 1.18, 1.33, 1.51, 1.62, 1.68, 2.02, 2.51, 2.73, 3.00, 4.00$ and $4.97$. The side ratios for the larger cylinders were: $B/A = 1.00, 1.62, 2.00, 2.50$ and $3.00$. The variations of the side dimensions over the lengths were within $\pm 0.015$ mm for the smaller cylinders and within $\pm 0.03$ mm for the larger ones.

2.2. End Conditions, Aspect Ratios and Blockages

For the smaller cylinders ($A = 4$ mm) the supporting plates were 480 mm apart and thus the aspect ratio $L/A$ was about 120 ($L/B > 25$). To improve the end conditions, the larger cylinders ($A = 20$ mm) were equipped with 5 mm thick circular end plates having a diameter of about $8B$. In a pre-study similar end plates with diameters $\sim 4B$ were tested. For $B/A = 1$ and $B/A = 1.62$, respectively, negligible effects of this change in relative end plate diameter were found. However, for $B/A = 2.5$ and 3, respectively, there was significantly less suction in the separated regions at around $\alpha = 90^\circ$ with the smaller end plates as compared with the larger ones. For the larger cylinders the maximum distance between the end plates was 100 cm (aspect ratio $L/A \approx 51, L/B > 17$). The influence of aspect ratio was tested for the case ($B/A = 1, \alpha = 0^\circ$). No changes on the averaged base pressure were found when cutting the aspect ratio down to $L/A = 25$. At an aspect ratio of $L/A = 10$, however, there was a decrease in the base pressure coefficient for $Re < 6 \times 10^3$ compared to the other two aspect ratios whereas no changes were found at
higher $Re$. At the lowest Reynolds number with these cylinders, i.e. $Re \approx 3 \times 10^4$, the difference was about 6%. It was suggested from the pre-study that the relative end plate diameter $D_{EP}/D$ was an important parameter, at least for aspect ratios $L/D$ less than about 20, $Re \sim 10^4$ and for $D_{EP}/D < 8$, in agreement with [16].

The blockages for the larger cylinders were within 1.5% to 4.7% whereas the maximum blockage for the smaller cylinders was about 1.6% (model width in relation to the height of the working section). Several schemes for blockage-correction at these rather low blockages are applicable, see e.g. [17, 18]. However, the results are given without any corrections.

2.3. Spectral Analysis and Pressure Measurements

The wake frequencies were measured with a standard single hot wire positioned in the outer parts of the near-wake shear layers. In each case the peak frequency was calculated from the frequency given by the arithmetic mean of the two frequencies where the spectral level was $3dB$ lower than the peak value. The difference between these two frequencies was used as a measure of the bandwidth of the peak. The analyzing bandwidths were less than 0.06% times $U_o/A$ and the mean spectra were averaged over at least 64 individual spectra. The uncertainty in the Strouhal number was estimated to be less than 1% at free stream velocities greater than 3 m/s.

Local wall pressure measurements at mid-span were carried out for the larger cylinders. On the cylinder with $B/A = 2$ one tap was at the centre of the short side “1” and one at the centre of the longer side “2” (Figure 1a). The remaining cylinders had five pressure taps on side “1”: one at the centre and the others at 2 and 5 mm from the corners, respectively. On side “2” for $B/A = 1.62, 2.5$ and 3 the number of taps were 7, 11 and 13, respectively. One was at the centre and the others symmetrically distributed 2, 5, 10, 15, 20 and 25 mm from the corners, respectively (2, 5 and 10 mm for $B/A = 1.62$ etc.). To avoid interference, the taps were positioned along a line 60° from the axis of the cylinders. The inner diameter of the taps were 0.8 mm. The mean pressure coefficient was defined as $C_P = (P - P_r)/(\rho U_o^2/2)$, where the reference pressure $P_r$ was taken as the static pressure from the Pitot-static tube. The minimum integration time was 20 seconds/tap. The free stream velocity was measured before and after the scanning through the wall taps and for each angle of attack $\alpha$. The uncertainty in the measured pressure coefficients was estimated to be less than 0.01.

3. RESULTS AND DISCUSSION

3.1. Angles of attack $\alpha = 0^\circ$ and $\alpha = 90^\circ$

Strouhal numbers and drag coefficients vs $H/D$ are shown in Figure 2. A general agreement with previous investigations was found [1, 2, 7, 9, 17, 19]. The most notable feature of Figure 2 is that in the range $H/D = 0 - 1$, $C_d$ undergoes significant changes while $St$ changes very little, whereas the reverse is true in the range $H/D = 2 - 3$. At around $H/D = 0.6$ the drag coefficient reaches a maximum. The present value for $H/D = 0.62$ at $Re = 13 \times 10^3$ was $C_d = 2.80$ (Table 1) as compared to the value 2.94 as given in [2] ($Re \sim 50 \times 10^3$). As shown in Figure 3b the vortex shedding at this “Golden Section” side ratio was extremely powerful with very strong curvatures of the near-wake separating shear layers. In addition, there is a non-uniformity of the
Figure 2. Strouhal number (a) and drag coefficient (b) vs $H/D$ at low free stream turbulence.

Figure 3. Surface pressure distributions and smoke-wire flow visualizations. (a) $B/A = 3$, $\alpha = 0^\circ$, (b) $B/A = 1.62$, $\alpha = 90^\circ$. Dotted lines refer to $C_P = 0$ whereas thick lines (cylinder surface) refer to $C_P = 1$. Arrows indicate resultant force coefficient with the same scaling as for the pressure coefficient $C_P$. Pressure distributions: $Re = 13 \times 10^3$, flow visualizations: $Re = 8 \times 10^3$. 
base pressure with a maximum suction at the centerline, in accordance with [2]. As demonstrated by Nakamura et al. [20, 21] the critical side ratio is reduced by the addition of small scale free stream turbulence as well as by transverse vibration especially at resonant conditions. As discussed in [2] the increase in $C_d$ up to the critical section is due to a reduction in the base cavity due to progressive interaction between the separating shear layers. Beyond the critical side ratio the influence from the rear corners forces the vortices to again form further downstream with the result of an increase in the base pressure i.e. a decrease in $C_d$. This shear layer/edge direct interaction [21] eventually results in a reattachment-type of flow with a jump to a higher Strouhal number and with a pressure distribution on the side face characterized by a plateau of low pressure followed by a recovery to a higher pressure somewhere near the base (Figure 3a, left).

Experimental as well as numerical [22, 23, 24, 25, 26] data on $St$ vs $Re$ at $\alpha = 0^\circ$ for some side ratios are shown in Figure 4. Multiple Strouhal numbers were sometimes found within certain Reynolds number ranges and for ($B/A = 2-3, \alpha \approx 0^\circ$), see also Figure 2a. In most cases, the peaks in the spectra were then rather broad-banded indicating a weak and possibly unstable vortex shedding (whenever greater than 2% the bandwidth of the peaks are indicated as vertical bars). A general agreement with the experimental Strouhal number data of Okajima et al. [3, 27] and Igarashi [28, 29] at $\alpha = 0^\circ$ was found. However, in some ranges of $Re$ the number of Strouhal peaks differed somewhat. The reason for this might be due to differences in the experimental set-ups. Also, in the near-wake region, the
appearance of the peaks showed a variation with measurement position. As evident from Figures 2a and 4 the change-over to reattachment-like flow has a complex dependence on the Reynolds number. The smoke-wire visualization in Figure 3a shows that the separated flow at \( H/D = 3, \Re = 8 \times 10^3 \) becomes unstable in the presence of the trailing-edge corner. However, at this instant, the flow does not seem to be fully reattached. Nevertheless, the spectral density for this case showed a single peak at \( St = 0.165 \) indicating a reattachment-like flow. As shown in [30] this flow probably exhibits periodic reattachment/detachment at the Strouhal frequency.

Table 1
Aerodynamic data for some investigated cases \((A = 20 \text{ mm})\)

<table>
<thead>
<tr>
<th>(B/A)</th>
<th>(\alpha_o)</th>
<th>(Re_D)</th>
<th>(C_d)</th>
<th>(St)</th>
<th>(C_{Y,\alpha})</th>
<th>(C_{M,\alpha})</th>
<th>(C_{P_s}^{m})</th>
<th>(C_{P_b}^{m})</th>
<th>(\alpha_R)</th>
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<td>38 \times 10^3</td>
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<td>0.131</td>
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<td>0.20</td>
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<td>1.41</td>
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<td>32 \times 10^3</td>
<td>2.22</td>
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<td>0.26</td>
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<tr>
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<td>90</td>
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<td>0.128</td>
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<td>4.52</td>
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<tr>
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<td>0.162</td>
<td>-25.8</td>
<td>11.9</td>
<td>0.93</td>
<td>0.49</td>
<td>0.47</td>
</tr>
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</table>

\(C_{Y,\alpha}\) and \(C_{M,\alpha}\) are the derivatives, with respect to \(\alpha\) around \(\alpha = \alpha_o\), of the vertical force and moment coefficients, respectively. Subscripts “s” and “b” refer to side and base surfaces whereas superscripts “m” and “c” refer to surface mean and surface centre values, respectively. \(\alpha_R\) is the angle from \(\alpha_o\) where reattachment flow was indicated (see text).

3.2. Angles of attack \(\alpha = 0^\circ - 90^\circ\)

The most dominant Strouhal numbers as a function of the angle of attack at various side ratios are shown in Figures 5 and 6a. Within central bands of \(\alpha\), i.e. the fully separated
regions, the Strouhal number was approx. constant at a level $St = 0.17$. At other angles, however, the flow showed a large influence of the side ratio due to e.g. reattachment. Also shown in Figure 6 are the lift, drag and moment coefficients, respectively. Some further data from these measurements with $A = 20$ mm are compiled in Table 1. In particular, the last column gives an indicated reattachment angle $\alpha_R$ – the actual values being taken out from the variation of the lift coefficient (Figure 6b). It was defined as the angle from $\alpha_o$, with increasing $\alpha$, where $C_\ell$ first reaches a local minimum after decreasing from zero at $\alpha = \alpha_o$. With this definition, no reattachment angle was indicated for the case ($B/A = 3$, $\alpha_o = 0^\circ$). However, as stated earlier, reattachment-like behaviour was for this case indicated at zero angle of attack i.e. $\alpha_R = 0^\circ$. The agreement with previous reported reattachment angles was satisfactory (e.g. [18]). The sign of the first derivative of the vertical force coefficient indicated that only three cases in Table 1 were aerodynamically stable against vertical vibrations in the quasi-steady limit. All cases showed a decrease in the moment coefficient with increasing $\alpha$ around $\alpha_o$ indicating stable postures [31] although being potentials for torsional galloping [13].

3.3. Galloping Response: Square Cylinder

From the variation of the force coefficient with angle of attack for the square cylinder, see Figure 7a, the limiting galloping response for $\alpha_o = 0^\circ$ was calculated (Figure 7b). The
Figure 7. (a) Transverse force coefficient vs angle of attack and (b) galloping response for \( \alpha = 0^\circ \) \( (\dot{a} = nY_S/(\beta A)) \) where \( Y_S \) is the amplitude of limiting oscillations; \( \dot{U} = nU_r/\beta \). Square section \( B/A = 1 \).

Final calculations were carried out with a 4th-order Runge-Kutta method using a time step of 0.05 times the period of oscillation and for a damping coefficient \( \beta = 0.05 \). The experimental data of \( C_{FY} \) vs \( \tan \alpha \) up to \( \alpha = 17^\circ \) were least-square-fitted to a 7th-order odd function polynomial, including second order power terms, where the first coefficient \( A_1 = C_{Y,\alpha} \) was forced to the value as given in Table 1. As shown in [12] the response at low \( nA_1 \) with the scaling as in Figure 7b depends on the fitted aerodynamic constants only. The increase in \( Re \) from \( 5 \times 10^3 \) to \( 13 \times 10^3 \) resulted in (i) an approx. 25% increase in \( A_1 \), (ii) a slight movement towards the origin for the maximum in \( C_{FY} \) associated with reattachment and (iii) a less distinguished plateau at around \( \alpha = 5^\circ \). The effects on the response, due to this increase in \( Re \), were limited to rather low \( \dot{U} \). Apart from the increase in the initial galloping speed there was also a reduction in the size of the hysteresis region (upper branch for decreasing velocities and vice versa). This sensitivity to Reynolds number might explain some earlier discrepancies between calculations and experiments (e.g. [11]).

4. CONCLUSIONS

New and additional experimental data on Strouhal numbers and other aerodynamic coefficients have been collected for a wide range of Reynolds number, different side ratios and angles of attack \( 0^\circ - 90^\circ \). Multiple wake frequencies were sometimes found at small angles of attack, within certain Reynolds number ranges and with side ratios \( B/A = 2 - 3 \). For intermediate angles of attack with fully separated flow the Strouhal number and drag coefficient based on the projected width, respectively, were approximately constant. Elsewhere, the flow showed a large influence of both angle of attack and side ratio due to e.g. reattachment and shear layer/edge interactions. When applicable the present data were in good agreement with existing information. For the square cylinder the measured pressure forces provided input for calculations of the limiting response in the quasi-steady limit of galloping oscillations in the plunging mode.
The support from the Swedish Research Council for Engineering Sciences (TFR) is gratefully acknowledged.

5. REFERENCES