Volumetric Pumps and Compressors
BMEGEVGAG04

Homework, sample solution

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Contents

1 Input data

- Design a piston pump based on the data below. The result of the design are (a) piston area, (b) stroke and (c) driving motor revolution number and power. The revoltuion number should be max. 200 rpm.
- Provide a minimal connecting rod diameter to avoid buckling!
- Assuming $\delta = 5\%$ allowed pulsation level, design a pulsation dampener (nominal gas volume V_0). Use the mean system pressure as mean pressure and isothermal process.
- Plot the following diagrams for two periods: $Q_{pump}(t)$, $Q_{PD}(t)$, $V_{air,PD}(t)$ and $p_{air,PD}(t)$. (PD = pulsation dampener)
- Assuming ambient tank pressure and H geodetical height difference between the fluid surface and the pump suction side, L pipe length (see the table below) and $\lambda = 0.02$ friction factor, find the minimum pipe diameter required to avoid cavitation. Choose the next standard diameter.

Type: triplex, mean flow rate: 30 l/min, mean system pressure: 150 bar, H=1 m, L=1 m.

2 Solution

2.1 Main geometry

The expected flow rate is 30 litre/min, which is 10 litre/min for each piston (we have a triple pump). This gives $Q_{mean,piston} = 10\,l/min$, that is

$$Q_{mean,piston} = Asn = 10\frac{\text{litre}}{\text{min}} = \frac{D^2\pi}{4}sn.$$

Note that we have *three* free parameters, i.e. piston diameter D, the stroke s and the revolution number n. The same flow rate can be achieved with a large stroke and small piston ($s \leq D$) or with a large piston and small stroke $D \leq s$. For geometrical reasons, we fix $D/2 \approx s$ and $n = 100\,\text{rpm}$, which gives

$$10\frac{\text{litre}}{\text{min}} = \frac{D^2\pi}{4}\frac{D}{2}100\,\text{rpm} \quad \rightarrow \quad D = 63.384\text{mm}.$$

We choose the diameter to be $D = 65\text{mm}$, which gives $s = 30.1\text{mm}$. This gives $V_{stroke} = D^2\pi/4s = 0.1$ litre.

2.2 Buckling

We have to verify the piston rod against buckling. Assuming clutched ends ($k = 1$), the critical force is

$$F_{crit} = \frac{\pi^2 EI}{kL^2}, \quad \text{with} \quad k = \sqrt{\frac{1}{n}} \quad \text{and} \quad I = \pi\frac{d^4}{64}.$$

For a standard steel (ASTM A36), we have $E = 200$ GPa. The force loading the piston is

$$F = \Delta pA = 150\text{bar} \times \frac{65^2\pi}{4}\text{mm}^2 = 49.77\,\text{kN}.$$

As the rod length of the crankshaft mechanism as approx. $L = 3s$, the minimum required piston rod diameter is

$$I_{min} = \frac{F_{crit}kL^2}{\pi^2 E} = \frac{49.8\,10^3 \times 1 \times (3 \times 30.1\,10^{-3})^2}{\pi^2\,200\,10^9} = 2.1\,10^{-10} \quad \rightarrow \quad D_{rod,min} = 8\,\text{mm}.$$

Let us apply a safety factor of approx. 1.25, giving $D_{rod} = 10\text{mm}$.

2.3 Pulsation dampener

The main sizing equation for a pulsation dampener is:

$$V_0 = \frac{\nu V_{stroke}}{\frac{p_{pc}}{p_{sys}}\delta_{max}}.$$

The required pulsation factor is $\delta_{max} = 5\%$, the precharge pressure is 80% of the system pressure, while, for a triplex pump, we have $\nu = 0.009$. This gives

$$V_0 = \frac{0.009 \times \frac{\left(65\,10^{-3}\right)^2\pi}{4} 30.1\,10^{-3}}{0.8 \times 0.05} = 0.022\,\text{litre}.$$

Notice that this is a very small volume, as the pulsation level a triple piston pump low. If we were to design a duplex or single-piston pump, one would need a significant volume for the pulsation dampener.

2.4 Flow rate diagrams

The flow rate of the three pistons are

$$\begin{aligned}
Q_1(t) &= \max\left(0, Q_{max}\cos\left(\omega t\right)\right) \\
Q_2(t) &= \max\left(0, Q_{max}\cos\left(\omega t + \frac{1}{3}2\pi\right)\right) \\
Q_3(t) &= \max\left(0, Q_{max}\cos\left(\omega t + \frac{2}{3}2\pi\right)\right)
\end{aligned}$$

where $Q_{max} = \pi A_p s = 0,314$ litre and $\omega = 2\pi n = 10.47$ rad/s. The pump flow rate is $Q_{pump} = Q_1 + Q_2 + Q_3$ while the mean pump flow rate, i.e. the system flow rate is $Q_{sys} = 3 \times A_p s n$. Hence, the flow rate of the pulsation dampener is $Q_{pd} = Q_{sys} - Q_{pump}$.

After precharging, the initial volume and pressure of the gas in the pulsation dampener is V_0 and p_{pc}, which changes to p_{sys} and V_{sys} once it is connected to the system:

$$p_{pc}V_0 = p_{sys}V_{sys} \quad \rightarrow \quad V_{sys} = V_0\frac{p_{pc}}{p_{sys}} = 0.08\,\text{litre}.$$

Next, we wish to compute the pressure change of the gas, that is performed by means of isotherm process:

$$p(t) = p_{sys}\frac{V_{sys}}{V(t)}, \quad \text{where} \quad V(t) = V_{sys} + \int_0^t Q_{pd}(\tau)d\tau.$$

The instantaneous gas volume is computed approximately by

$$V(t_i + \Delta t) = V(t_i) + \int_{t_i}^{t_i+\Delta t} Q_{pd}(\tau)d\tau \approx V(t_i) + \frac{\Delta t}{2}\left(Q_{pd}\left(t_i\right) + Q_{pd}\left(t_i + \Delta t\right)\right),$$

and $V(0) = V_{sys}$. Δt is an appropriately chosen timestep, for example $T/20$ where T is the period of the shaft revolution. In our case, we have $T = 0.6$ s (n=100 rpm) and choose $\Delta t = T/20 = 0.03$ s.

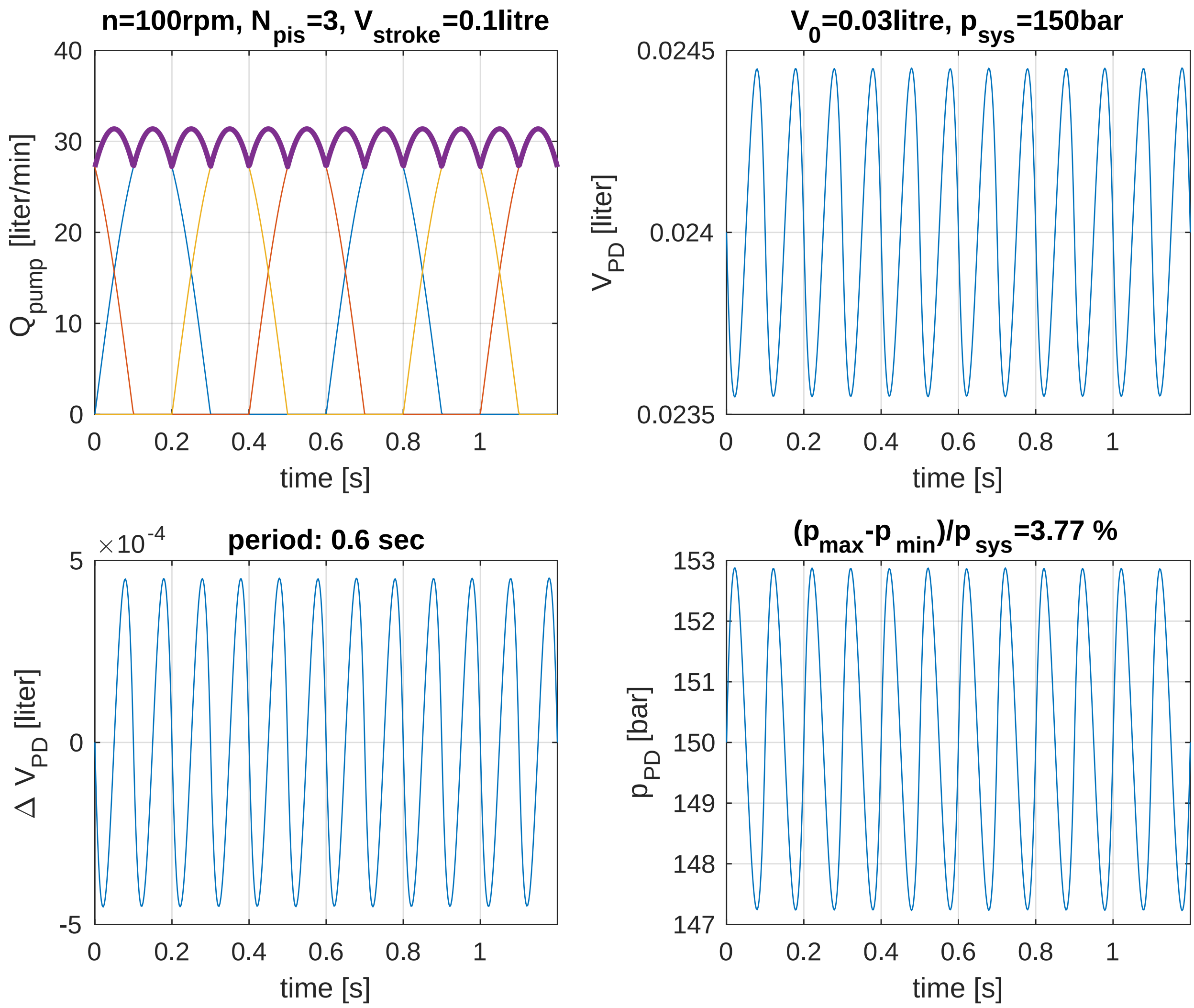

Figure 1: Flow rate diagram

2.5 Cavitation

The sizing equation is

$$p_{s,min} > p_t - \underbrace{\rho g h_s}_{p_{geo}} - \underbrace{\frac{\rho}{2}\frac{Q_{max}^2}{A_s^2}\left(1 + \lambda\frac{L}{D}\right)}_{p_{fric.}} - \underbrace{\rho L \frac{\omega Q_{max}}{A_s}}_{p_{accel.}}. \tag{1}$$

The actual numbers are

- $p_t = 1$ bar,
- $\rho g h_s = 0.0981$ bar,
- $Q_{max} = 31.4$ litre/min (based on the pump flow rate diagram),
- $\omega = 2\pi n = 10.47$ rad/s and
- the vapour pressure of water at room temperature is $p_v = 2.34$ kPa.

With the above numbers, the only remaining unknown is the suction side pipe diameter D_s

D_s,mm	$p_{geo.}$, bar	$p_{fric.}$, bar	$p_{accel.}$, bar	$p_{s,min}$, bar
10	0.0981	0.666	0.698	-0.462
15	0.0981	0.102	0.310	0.310
20	0.0981	0.028	0.174	0.700
30	0.0981	0.005	0.078	0.820

As it can be seen, the minimum pressure above $D_s = 15$mm is larger than the vapour pressure, hence we choose, say, $D_s = 30$mm.