1. MEASUREMENT

MEASUREMENT OF REVOLUTION NUMBER, MOMENT OF INERTIA AND FRICTION TORQUE

1. Introduction

THE AIM OF THE MEASUREMENT is to determine the relationship between the ***n*** revolution number and the ***t*** elapsed time (***n(t)*** diagram) from time point of switching off the electromotor until the full stop. Further task is the determination of the M_s friction torque in the function of revolution number, which slows down the rotation of the shaft. In order to finish the tasks successfully, it is necessary to be familiar with the basic definitions (revolution number, moment of inertia, ***n(t)*** diagram) and their methods of measurement.

SHORT DESCRIPTION OF THE MEASUREMENT: To determine the $M_s(n)$ friction torque of the electromotor depending on the revolution number, the ***n(t)*** diagram is needed. After switching off the electromotor, the rotor of the motor continuously slows down due to the friction torque, and finally it stops. The ***n(t)*** function can be determined with the help of a revolution number transducer mounted on the rotor shaft and with a computer.

Then the friction torque is determined in the points of ***n(t)*** diagram using Newton's second law:

$$M_f = \Theta\varepsilon,$$

where ε is the angular acceleration obtained from the ***n(t)*** diagram, and Θ is the moment of inertia, which is measured with the help of another, but the same type of electromotor.

SUMMARY:

- Measurement of ***n(t)*** diagram and Θ moment of inertia. These measurements can be performed parallelly.
- Determination of ε angular acceleration with the help of ***n(t)*** diagram.
- Calculation of M_s friction torque.

2. The *n(t)* diagram and its measurement

To keep the machine part mounted on the motor shaft at constant speed, the drive motor must cover all braking torque on the rotating part. This braking torque in our case coming from the friction of the bearings (the braking torque of the air resistance acting on the surface of the rotor is neglected).

When the motor is switched off the tested machine part will rotate with a reducing speed due to the friction until it stops.

If the momentary value of the ***n*** revolution number is measured during the decelerating rotation, and ***n*** is plotted against the time, man can obtain the ***n=n(t)*** diagram. The ***n(t)*** diagram shows the revolution number of the shaft in the function of time.

The measurement setup is shown by Figure 1. The shaft of the electric motor "1" is connected to the „2” revolution number transducer giving voltage signal proportional to the revolution number. Its analog signal is processed by the computer "3". The main steps of signal processing are digitizing, filtering and plotting as a function of time. During the measurement, the motor is switched off and we wait until it stops (approx. 35 s). Then, the ***n=n(t)*** plot appears on the screen.

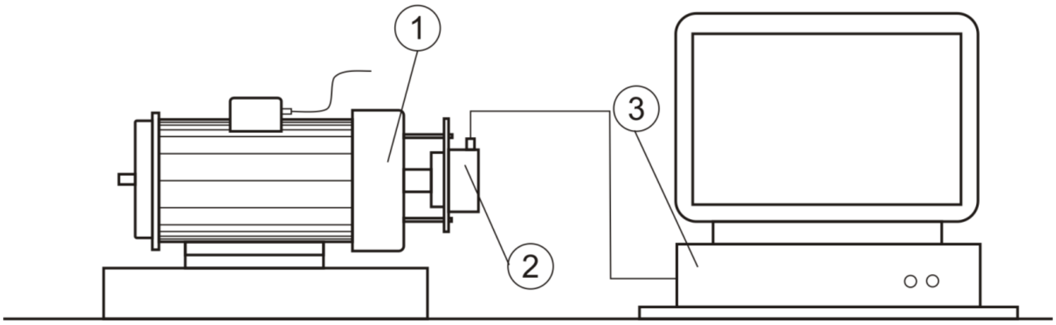

Figure 1. Schematic drawing of the setup for measuring n(t) diagram

(Parts of the rig: 1: electromotor; 2: revolution number transducer*; 3: computer)*

A typical shape of ***n(t)*** function can be seen on Figure 2. It can be seen that the machine part stops after t_0 time since the motor was switched off. With the help of the measurement recording program, the ***t*** time, the ***n*** revolution number and the ε angular acceleration can be read at any arbitrary point (the angular acceleration is calculated automatically by the program).

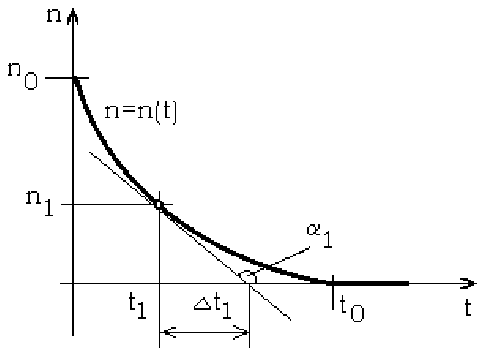

Figure 2. ***n(t)*** diagram

The ***n(t)*** diagram obtained during measurement has to be drawn on a millimetre paper with the help of the uniformly distributed 12 measuring points read by the program. (The number of points may vary depending on the group size) After the measurement, the measuring points are written on the blackboard so the ***n(t)*** diagram can be drawn immediately.

3. The moment of inertia, and its measurement

3.1. The moment of inertia

Let us consider the case when a ring of mass ***m*** moves on a circular path with radius ***r*** (e.g. bicycle wheel with negligible mass spokes). Then its moment of inertia is

$$\Theta = mr^2.$$

The moment of inertia of a cylinder respect to own rotation axis is

$$\Theta = \frac{1}{2} m r^2.$$

where ***m*** is the mass and ***r*** is the radius of the cylinder. In general cases the moment of inertia depends on the shape of the rotating body, on the mass distribution and as well on the distances of the masses from the centre of rotation. It can be determined with mathematical tools. If an irregular shaped body is considered, it is needed to divide it into small masses Δm_i lying on radius of r_i. Then the moment of the inertia of a small part is

$$\Delta\Theta_i = \Delta m_i r_i^2.$$

The moment of inertia of the full body can be calculated as a sum of these small parts:

$$\Theta_i = \sum \Delta m_i r_i^2.$$

It is important to note that, the different moments of inertia are additives. It means, that if an m_2 mass on r_2 radius mounted on a cylinder with m_1 mass and r_1 radius ($r_2 < r_1$), the resulting moment of inertia is

$$\Theta = \frac{1}{2} m_1 r_1^2 + m_2 r_2^2.$$

3.2. The moment of inertia of the rotor

To evaluate our measurements, the moment of inertia of the rotor (Θ) is needed. According to left hand side of **Figure 3** we turn our rotor into a physical pendulum by mounting an additional mass (cylinder of uniform mass-distribution) on the rotating part at a distance of ***d*** from the rotation axis.

The physical pendulum is a rigid body having extension and hanging on ***A*** rotation axis, see middle of **Figure 3**. Later, the rotor-additional mass pendulum will be referred as physical pendulum. The additional mass is a solid cylinder with ***d=2r*** diameter, and its axis is in the distance of ***e*** from the rotation axis. Let us neglect the mass of the metal plate helping the mounting of cylinder to the rotor, therefore the metal plate has no role in moment of inertia calculation. Due to the additional mass the centre of gravity of the physical pendulum is displaced from point ***A*** to point ***S***.

The moment of inertia of the additional mass builds from the $\frac{1}{2} m_a r^2$ value calculated respect to its own axis, and from the $m_a e^2$ so called Steiner part (the Steiner-law will be described in subject Dynamics later) coming from the fact that, the own axis is displaced from the rotation axis with ***e*** distance. Then, as a result

$$\Theta_\mathrm{a} = \frac{1}{2} m_p r^2 + m_p e^2.$$

The resulting moment of inertia of the rotor-additional mass physical pendulum (right hand side of **Figure 3**.) is the sum of rotor's and additional mass' moment of inertia:

$$\Theta_\mathrm{p} = \Theta + \Theta_a.$$

If the moment of inertia of the physical pendulum could be determined, then rotor's one could be obtained easily from

$$\Theta = \Theta_\mathrm{p} - \Theta_\mathrm{a} = \Theta_\mathrm{p} - \frac{1}{2} m_a r^2 - m_a e^2.$$

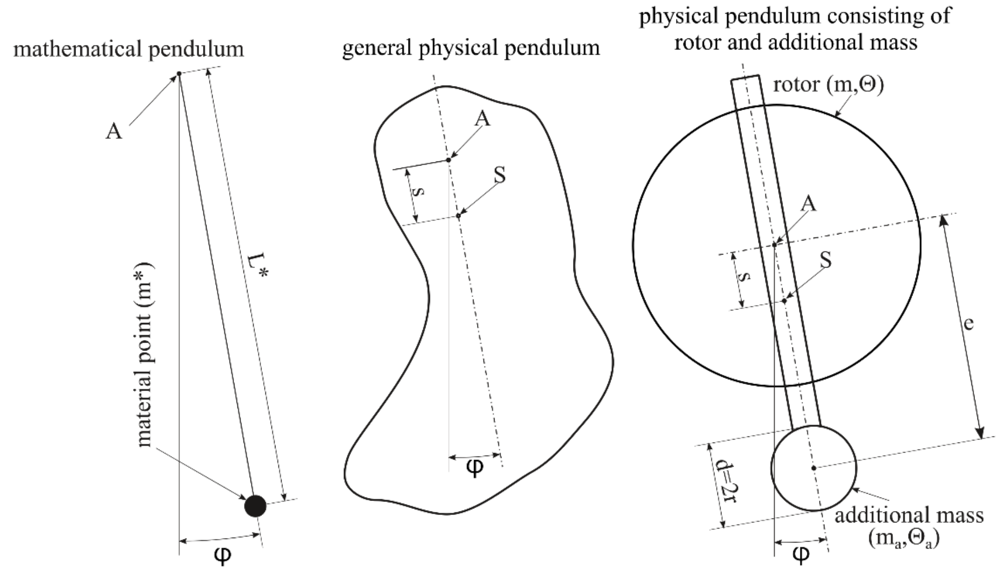

Figure 3. The comparison of physical and mathematical pendulum

Because of the fact that the $\boldsymbol{T_p}$ time period of oscillation of physical pendulum depends on the moment of inertia, the measurement of the period of oscillation is necessary. The problem is that there is no formula for calculation of time period in the case of complex geometry. Thus, the physical pendulum is compared to an equivalent mathematical pendulum, see left hand side of **Figure 3**. The $\boldsymbol{L^*}$ reduced length of the mathematical pendulum must be set so that the $\boldsymbol{\varepsilon_p}$ angular acceleration of the physical pendulum is the same as $\boldsymbol{\varepsilon_m}$ angular acceleration of the mathematical pendulum. In this case the time periods will be the same. Applying Newton's second law the angular accelerations are

$$\varepsilon_p = \frac{M_p}{\Theta_p} = -\frac{(m+m_a)\cdot g\cdot s\cdot \sin\varphi}{\Theta_p},$$

$$\varepsilon_m = \frac{M_m}{\Theta_m} = -\frac{m^*\cdot g\cdot L^*\cdot \sin\varphi}{m^* {L^*}^2} = -\frac{g\cdot\sin\varphi}{L^*}.$$

From the equality of the two angular acceleration the reduced length is

$$L^* = \frac{\Theta_p}{(m+m_a)\cdot s}.$$

It is worth to note that there are only the parameters of physical pendulum on the left-hand side of the equation.

Based on the equality of the time periods and the formula of time period of mathematical pendulum the following expression can be written:

$$T = T_m = 2\pi\sqrt{\frac{L^*}{g}} = T_p = 2\pi\sqrt{\frac{\Theta_p}{(m+m_a)\cdot s\cdot g}}.$$

Since it is difficult to calculate with the $\boldsymbol{s}$ distance between the $\boldsymbol{S}$ centre of gravity and point $\boldsymbol{A}$, the torque equilibrium on the point $\boldsymbol{A}$ is applied ($\boldsymbol{e}$ length can be measured):

$$(m+m_a)\cdot s\cdot g = m_{as}\cdot e\cdot g.$$

Finally, the moment of inertia of the physical pendulum can be already determined by measuring the oscillation time and the parameters of the additional mass:

$$\Theta_p = m_a \cdot e \cdot g \cdot \left(\frac{T}{2\pi}\right)^2.$$

3.3. The process of measuring moment of inertia

The measurement will be made by groups of 3 members. Each group will measure the rotor's moment of inertia independently from each other, which means 4 values in the case of 4 groups. It will be seen, that the values fluctuate, therefore the 4 values will be averaged and the $\boldsymbol{\Theta_M}$ mean value will be used to calculate friction torque.

This means that as long as all the groups do not measure the inertia torque, we cannot calculate the friction torque.

Figure 4 shows the measure rig and its schematic drawing. It can be seen that the additional mass "2" (see **Figure 3**) was attached to the motor shaft.

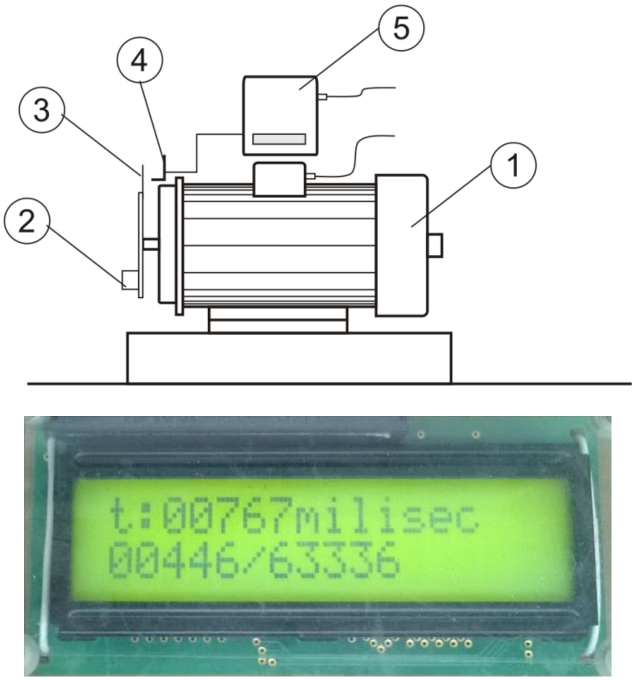

Figure 4. Measurement of moment of inertia

(Above: Parts of the measurement rig:1: electromotor; 2: additional mass; 3: plate;
4: magnetic impulse sender; 5: time period measuring device
Below: screen of the time period measuring device)

Each group selects a specific additional mass and writes down its parameters (diameter ***d***, mass $\boldsymbol{m_a}$). The additional mass is fixed to the bored metal plate by a screw. Remember to measure the ***e*** distance before fixing. The assembled physical pendulum is rotated until the scratched mark on the plate "3" is vertical (5-degree rotation).

The magnet on the disc passes before the "4" magnetic impulse sender, which gives a signal, taking into account the direction of the passage (always one per oscillation). From this, the period of the oscillation can be measured directly and can be read from the display "5". Knowing the ***T*** time, the moment of inertia can be calculated as above.

4. Friction torque

If all groups have completed the moment of inertia measurement and have obtained the Θ_M mean moment of inertia, furthermore everybody is ready with the draw of n(t) diagram , then the measurement can be continued by calculating and depicting the friction torque on a separate millimetre paper.

Of course, you do not have to calculate the friction torque at all 12 measuring points. Before starting the seminar, everyone will get a serial number and will only work with data corresponding to the appropriate number from the table drawn on the blackboard. Remember, at the beginning of the seminar we make the ***n(t)*** diagram and record 12 uniformly distributed measurement points on the blackboard containing ***t*** time, ***n*** revolution number and ε angular acceleration.

The friction torque can be determined using Newton's II. law:

$$M_f = \Theta_M \varepsilon.$$

After calculating the result, the obtained values are written into the table drawn onto the blackboard.

4.1. Checking angular acceleration

Check at each measuring point (note the time with t_1 and the corresponding revolution number n_1) by drawing a tangent line at timepoint t_1 in ***n=n(t)*** diagram and calculating the slope of that line. see Figure 2. The slope of the tangent line gives the value of the angular acceleration at time t_1 (it is negative because it slows down):

$$\varepsilon_1 \approx \frac{\Delta\omega}{\Delta t} = \frac{2\pi \cdot \Delta n}{\Delta t} = -2\pi \frac{n_1}{\Delta t_1} = -2\pi \cdot \tan\alpha_1,$$

where

$$\tan\alpha_1 = \frac{n_1}{\Delta t_1}.$$

In the above formula the $\omega=2\pi n$ relationship between angular velocity and revolution number was used. Furthermore, we applied the fact that the angular acceleration is equal to the change of the angular velocity per time unit ($\varepsilon\approx\Delta\omega/\Delta t$). The angular acceleration calculated from the slope and read from the program should be nearly the same!

Preparation to the measurement

- Bring 2 pieces of millimetre paper in the size of A4, pencil, ruler, calculator.
- Before the measurement, we will check the proper preparation for the measurement, the knowledge and the correct application of the formulas with short theoretical and numerical examples. (e.g. sample questions on the website) Please note that there may be other questions in the test.
- Fill the lab report templates at home up to point 4 (points 5-8 will be detailed in the lesson).

Notes on measurement description or the whole measurement welcome to e-mail address of csizmadia@hds.bme.hu

Appendix

6. Revolution number and its measurement

6.1. The revolution number of the rotating shaft is the number of revolutions per unit time. Its notation is mostly ***n***. Its dimension is rev/s or rev/min (rpm), but often only 1/min is used. The relationship between ***ω*** angular velocity and the ***n*** revolution number is

$$\omega = 2 \cdot \pi \cdot n \left[\frac{rad}{s}\right]$$

From the point of view of the measuring concept the instruments measuring the revolution number can be divided into two groups:

- **speed indicators:** measure the average revolution number
- **tachometers:** measure the momentary revolution number

6.2. Speed indicators

a.) Measurements of small revolution number (up to 120-150 1/min) can be made with stopwatch and by counting revolutions with naked eye. When the mark on the rotating machine shaft gets to a certain place, we start the stop-watch and synchronously begin to count with 0. Having measured ***t*** time of some (***N***) revolutions the number of revolutions per unit time (***n***) can be determined:

$$n = \frac{N}{t} \left[\frac{1}{s}, \frac{1}{\min}\right],$$

If the speed is fluctuating, the latter relationship gives the average revolution number for the time. The longer time during the number of revolutions is counted, the more accurately the revolution number can be determined.

b.) For higher speed of rotation a special counting device must be used. One of the simplest of these is the so called **jumping-figure speed counter**. The rotating shaft of this device turns gears. One of them completes one revolution while the other rotates only 1/10, and so on. Reading the numbers uniformly painted from 0-9 on the cylinder jacket we get the number of revolutions. Such a device is used in kilowatt-hour meters, tape recorders, speedometers of cars etc. If the elapsed time is measured with a stopwatch, the average revolution number can be calculated.

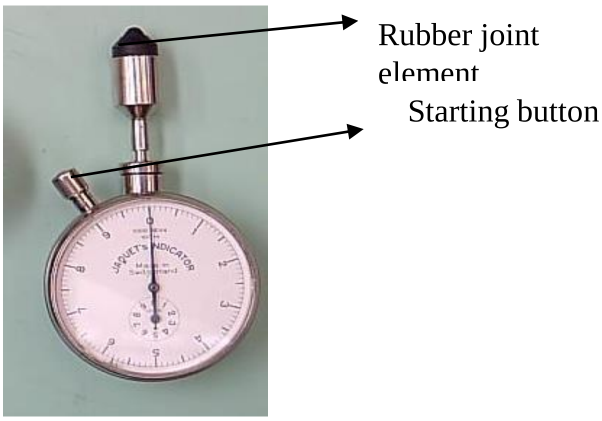

Figure 5. Jacquet indicator.

c.) **Speed indicator working with clockworks (Jacquet indicator, Fig. 5.)** counts the revolutions only for a fixed time, generally for 6 seconds. The time measuring device of the instrument connects its pointer with that shaft of the instrument which joints the rotating machine part for 6 seconds after pressing the starting bottom. After these six seconds there is no more connection which means at the same time the end of the measurement. On the dial the tenfold value of the counted revolutions i.e. rpm. (revolution per minute) can be read. This device is the so called Jacquet indicator. (Order of measurement: joint the shaft of the indicator to the shaft of the motor to establish connection between them, pressing starting button, release starting bottom, waiting around 8 second while the device measure, release the connection between shafts, read the value). With pressing the starting button, the instrument is zeroed and when it is let out the counting and the clockwork starts. Therefore, the buttons should be let out only when the joint piece rotates together with the shaft! It is expedient to zero the instrument only with the starting of the new measurement. The joint pieces are the same as seen in Fig.5.

d.) **Electric speed indicator** are composed of one or more markers giving voltage pulse and of a pulse counter operating on electric principle. The marker is generally a photocell in front of which a disc with gaps ensures one or more illuminations at each turning. The electric circuit is closed by the illuminated photocell. (Figure 6)
This counter can be placed far off the site of the measurement.
In order to increase accuracy and reduce measurement time it is advisable to increase the number of gaps.

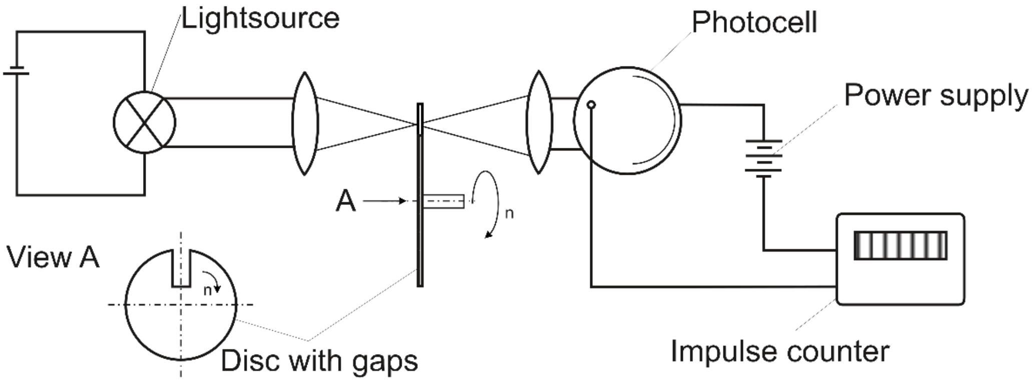

Figure 6. Electric speed indicator

e.) **Electric stroboscopes** illuminate the rotating or vibrating body with flashing lamps of variable and adjustable frequencies. If the frequency of the flashes corresponds to the speed of the rotating object, the object seems to be stationary. The frequency i.e. the revolution number can be read on the instrument. (Figure. 7.)

Figure 7. Stroboscope.

2.3. The most known **tachometer** is the dynamo, which work on electrical principle. This instrument produce voltage signal proportional to the revolution number. If the relationship between the voltage and revolution number is known (calibration), then the revolution number can be determined from the measured voltage signal.