Problem solving exercise 3

Problem 1

A single stage radial pump's volumetric flow rate is 0.055 $\frac{\text{m}^3}{\text{s}}$, the head is 45 m, and the speed of rotation is 1470 $\frac{1}{\text{min}}$. The hydraulic power loss is 2.5 kW, the mechanical power loss is 1.3 kW, the the disc friction coefficient is $\nu_{df} = 0.065$. The input power is 32 kW. Calculate the inner and useful power (P_{inner}, P_u), the head loss (h'), the leakage flow rate (Q_l)! Find the theoretical head (H_{th}) and theoretical volumetric flow rate (Q_{th})! Find the hydraulic (η_h), volumetric (η_v) and total (η) efficiency! The international best efficiency point (BEP) formula for pumps is the following:

$$\eta_{max} = 0.94 - 0.048 \cdot Q_{opt}^{-0.32} - 0.29 \cdot \log_{10}\left(\frac{n_q}{44}\right)^2. \tag{1}$$

Compare the efficiency from the formula above with the efficiency of the pump!

Solution

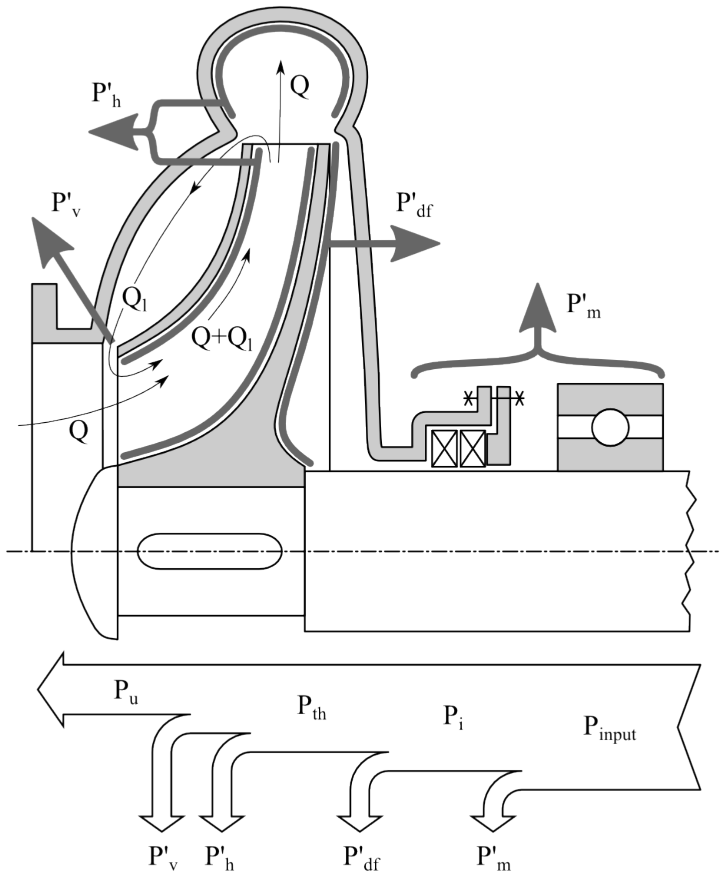

Figure 1: Losses in a pump

$$P_u = Q\rho g H = 0.055 \cdot 10^3 \cdot 9.81 \cdot 45 = 24.28 \text{ kW}$$
$$P_{in} = 32 \text{ kW}$$
$$P_{inner} = P_{in} - P'_{mech} = 32 - 1.3 = 30.7 \text{ kW}$$
$$P_{th} = P_{inner}(1 - \nu_{df}) = 30.7 \cdot (1 - 0.065) = 28.705 \text{ kW}$$
$$P'_h = Q\rho g h'$$
$$h' = \frac{P'_h}{Q\rho} = \frac{2500}{0.055 \cdot 10^3 \cdot 9.81} = 4.63 \text{ m}$$
$$H_{th} = H + h' = 45 + 4.63 = 49.63 \text{ m}$$
$$Q_{th} = \frac{P_{th}}{\rho g H_{th}} = \frac{28705}{10^3 \cdot 9.81 \cdot 49.63} = 0.059 \ \frac{\text{m}^3}{\text{s}}$$
$$\eta_h = \frac{H}{H_{th}} = \frac{45}{49.63} = 0.907$$
$$\eta_v = \frac{Q}{Q_{th}} = \frac{0.055}{0.059} = 0.932$$
$$Q_l = Q_{th} - Q = 0.059 - 0.055 = 0.004 \ \frac{\text{m}^3}{\text{s}}$$
$$\eta_m = \frac{P_{inner}}{P_{in}} = \frac{30.7}{32} = 0.959$$
$$\eta = \frac{P_u}{P_{in}} = \frac{24.28}{32} = 0.7588$$

Checking if the product of the efficiencies gives back the total efficiency:

$$\eta = \eta_v \eta_h \eta_m (1 - nu_{df}) = 0.932 \cdot 0.907 \cdot 0.959(1 - 0.065) = 0.758$$
$$n_q = n\frac{Q^{0.5}}{H^{0.75}} = 1470\frac{0.055^{0.5}}{45^{0.75}} = 19.8$$
$$\eta_{max} = 0.94 - 0.048 \cdot Q_{opt}^{-0.32} - 0.29 \cdot \log_{10}\left(\frac{n_q}{44}\right)^2 =$$
$$= 0.94 - 0.048 \cdot 0.055^{-0.32} - 0.29 \cdot \log_{10}\left(\frac{19.8}{44}\right)^2 = 0.94 - 0.121 - 0.035 = 0.785 > 0.758.$$

Problem 2

The performance curve of a pump at the speed of rotation $n_1 = 1460\ \frac{1}{\text{min}}$ is $H_1 = 40 - 40000(\frac{\text{s}^2}{\text{m}^5})Q^2$. Using the law of affinity, calculate 5 points of the performance curve in the case when the speed of rotation is $n_2 = 2920\ \frac{1}{\text{min}}$ in the interval $Q_2\ \in [0.01,\ 0.05]$ with 0.01 spacing. Find the performance curve at the second speed of rotation, using the law of affinity ($H_2(Q) = ?$). Find the specific speed of the pump at the higher speed of rotation! At the design point (the point with the highest efficiency) the volumetric flow rate is $Q_{2,des} = 0.03$.

Solution

$$\frac{n_2}{n_1} = \frac{Q_2}{Q_1} = 2 \rightarrow \frac{H_2}{H_1} = \left(\frac{Q_2}{Q_1}\right)^2 = 2^2 = 4$$

Q_1 ($\frac{\text{m}^3}{\text{s}}$)	0	0.005	0.01	0.015	0.02	0.025
H_1 (m)	40	39	36	31	24	15
Q_1 ($\frac{\text{m}^3}{\text{s}}$)	0	0.01	0.02	0.03	0.04	0.05
H_1 (m)	160	156	144	124	96	60

Performance curve calculation

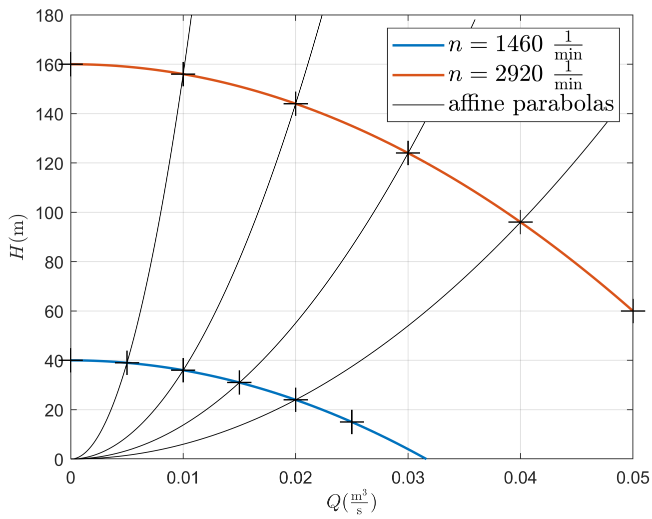

Figure 2: The law of affinity for the current problem

General form of a performance curve: $H_1 = A + BQ + CQ^2$ at n_1.
Using the law of affinity (omitting BQ):

$$H_2 = H_1\left(\frac{n_2}{n_1}\right)^2 = \left(\frac{n_2}{n_1}\right)^2\left(A + CQ_1^2\right) = \left(\frac{n_2}{n_1}\right)^2\left[A + CQ_2^2\left(\frac{n_1}{n_2}\right)^2\right] = A\left(\frac{n_2}{n_1}\right)^2 + CQ_2^2.$$

In the current problem:

$$H_2 = A\left(\frac{n_2}{n_1}\right)^2 + CQ^2 = 40\left(\frac{2920}{1460}\right)^2 - 40000Q^2$$

$$n_q = n_2\frac{Q^{0.5}}{H^{0.75}} = 2920\frac{0.03^{0.5}}{124^{0.75}} = 13.61$$

Problem 3

Calculate the required head of a pump that carries a volumetric flow rate $Q = 1200\ \frac{\mathrm{dm}^3}{\mathrm{min}}$ through the following pipeline. The diameter of the suction pipe is $D_s = 120$ mm, and the suction side loss coefficient is $\zeta_s = 3.6$; the pressure side pipe diameter is $D_p = 100$ mm, and the loss coefficient here (without the outlet loss!) is $\zeta_p = 14$. The pump comveys water from an open reservoir, which is 5 m below the base level, and the pressure side pipe end 25 m above the base level, where the water end up in a swimming pool, at which atmospheric pressure can be consivered. Draw the schematics of the pipeline!

Solution

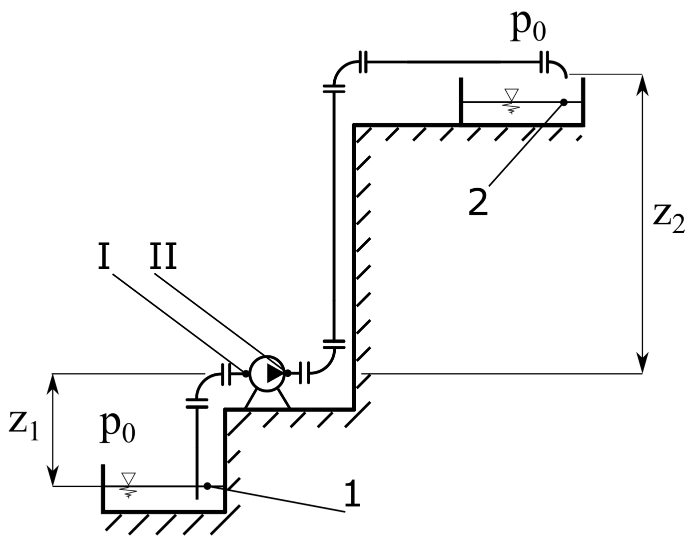

Figure 3: Drawing of the hydraulic system

Bernoulli eq. between points 1 and I (suction side): $e_1 = e_I + h'_s$. (e is the Bernoulli enthalpy in head)

Bernoulli eq. between points II and 2 (pressure side): $e_{II} = e_2 + h'_p$.

The suction side loss: $h'_s = \zeta_s \frac{v_s^2}{2g} = \zeta_s \frac{Q^2}{A_s^2 2g}$

The pressure side loss: $h'_p = \zeta_p \frac{v_p^2}{2g} = \zeta_p \frac{Q^2}{A_p^2 2g}$

The pumps head: $H = e_{II} - e_I = (e_2 - h'_p) - (e_1 - h'_s) = (e_2 - e_1) + h'_p + h'_s = \left(\frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2\right) - \left(\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1\right) + h'_s + h'_p$.

Using that $p_1 = p_0$, $p_2 = p_0$, $v_2 = v_p = Q^2/A_p^2$ and $v_1 = 0$, H can be simplified:

$$H = z_2 - z_1 + \overbrace{\frac{1}{2g}\frac{Q^2}{A_p^2}}^{\text{outlet loss}} + \overbrace{\zeta_s \frac{1}{2g}\frac{Q^2}{A_s^2}}^{h'_s} + \overbrace{\zeta_p \frac{1}{2g}\frac{Q^2}{A_p^2}}^{h'_p} = z_2 - z_1 + \frac{1}{2g}\left(\frac{1}{A_p^2} + \zeta_s \frac{1}{A_s^2} + \zeta_p \frac{1}{A_p^2}\right) Q^2$$

$A_s = \frac{D_s^2\pi}{4} = \frac{0.12^2\pi}{4} = 0.01131\ \mathrm{m}^2$
$A_p = \frac{D_p^2\pi}{4} = \frac{0.1^2\pi}{4} = 0.007854\ \mathrm{m}^2$
Substituting to the formula of the pumps head:

$$H = 25 - (-5) + \frac{1}{2 \cdot 9.81}\Big(\frac{1}{(7.854 \cdot 10^{-3})^2} + 14\frac{1}{(7.854 \cdot 10^{-3})^2} + 3.6\frac{1}{(1.131 \cdot 10^{-2})^2}\Big)0.02^2 = 35.53\ \mathrm{m}. \quad (2)$$